

1.1 # 79

difference quotient $\frac{f(x+h) - f(x)}{h}, h \neq 0$

$$f(x) = \frac{1}{x^2}$$

$$f(x+h) = \frac{1}{(x+h)^2} = \frac{1}{(x+h)(x+h)} = \frac{1}{x^2 + 2hx + h^2}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x^2 + 2hx + h^2} - \frac{1}{x^2}}{h}$$

need common denom
CO: $x^2(x^2 + 2hx + h^2)$

$$= \frac{\frac{x^2}{x^2} \cdot \frac{1}{x^2 + 2hx + h^2} - \frac{1}{x^2} \cdot \frac{x^2 + 2hx + h^2}{x^2 + 2hx + h^2}}{h}$$

$$= \frac{\frac{x^2}{x^2(x^2 + 2hx + h^2)} - \frac{x^2 + 2hx + h^2}{x^2(x^2 + 2hx + h^2)}}{h}$$

$$= \frac{\frac{x^2 - (x^2 + 2hx + h^2)}{x^2(x^2 + 2hx + h^2)}}{h}$$

subtract numerators
distribute the negative

$$= \frac{\frac{x^2 - x^2 - 2hx - h^2}{x^2(x^2 + 2hx + h^2)}}{h}$$

cancel terms

$$= \frac{\frac{-2hx - h^2}{x^2(x^2 + 2hx + h^2)}}{h}$$

put the "h"
over one

$$= \frac{h(-2x-h)}{x^2(x^2+2hx+h^2)} \cdot \frac{1}{h}$$

$$= \frac{\cancel{h}(-2x-h)}{x^2 \cancel{h}(x^2+2hx+h^2)}$$

$$= \frac{-2x-h}{x^2(x^2+2hx+h^2)}$$

$$\boxed{= \frac{-2x-h}{x^4+2hx^3+h^2x^2}}$$

reciprocate
the denominator
and multiply
it by numerator
factor out "h"
from numerator
"h" cancels

distribute the
"x²" in
denominator