

SUMMARY

- Graph of a Function** The collection of points (x, y) that satisfies the equation $y = f(x)$.
- Vertical-Line Test** A collection of points is the graph of a function provided that every vertical line intersects the graph in at most one point.

1.2 Assess Your Understanding

'Are You Prepared?' Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

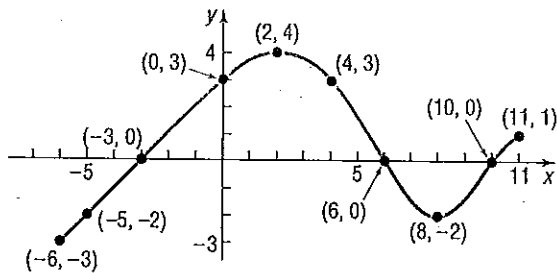
- The intercepts of the equation $x^2 + 4y^2 = 16$ are _____ . (p. 12)
- True or False** The point $(-2, -6)$ is on the graph of the equation $x = 2y - 2$. (pp. 9–10)

Concepts and Vocabulary

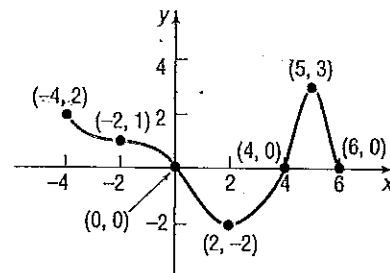
- A set of points in the xy -plane is the graph of a function if and only if every _____ line intersects the graph in at most one point.
- If the point $(5, -3)$ is a point on the graph of f , then $f(\underline{\quad}) = \underline{\quad}$.
- Find a such that the point $(-1, 2)$ is on the graph of $f(x) = ax^2 + 4$.
- True or False** A function can have more than one y -intercept.
- True or False** The graph of a function $y = f(x)$ always crosses the y -axis.
- True or False** The y -intercept of the graph of the function $y = f(x)$, whose domain is all real numbers, is $f(0)$.

Skill Building

- Use the given graph of the function f to answer parts (a)–(o).
- Use the given graph of the function f to answer parts (a)–(o).



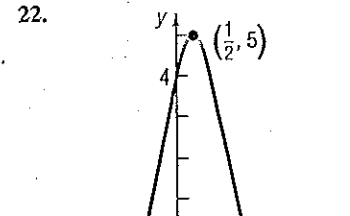
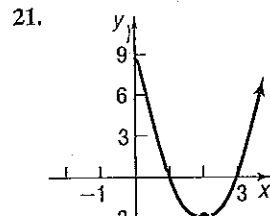
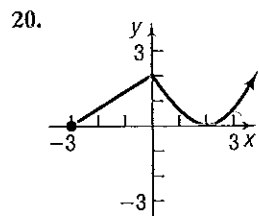
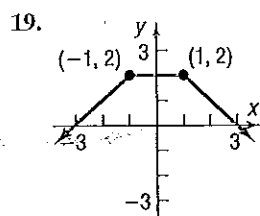
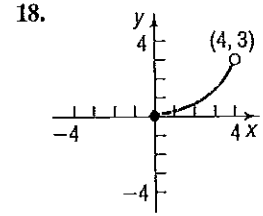
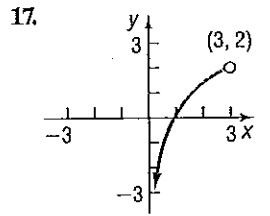
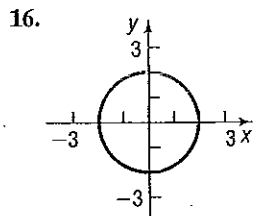
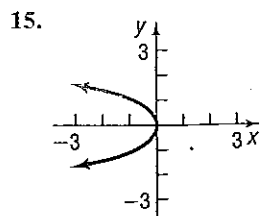
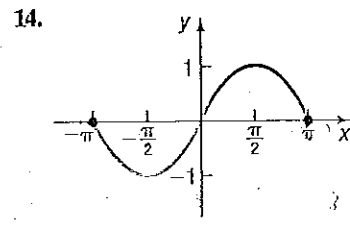
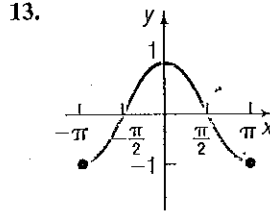
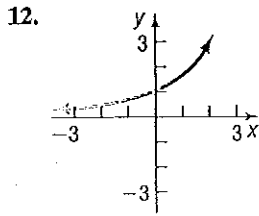
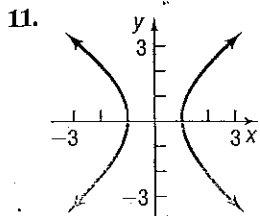
- Find $f(0)$ and $f(-6)$.
- Find $f(6)$ and $f(11)$.
- Is $f(3)$ positive or negative?
- Is $f(-4)$ positive or negative?
- For what values of x is $f(x) = 0$?
- For what values of x is $f(x) > 0$?
- What is the domain of f ?
- What is the range of f ?
- What are the x -intercepts?
- What is the y -intercept?
- How many times does the line $y = \frac{1}{2}$ intersect the graph?
- How many times does the line $x = 5$ intersect the graph?
- For what values of x does $f(x) = 3$?
- For what values of x does $f(x) = -2$?
- What are the zeros of f ?



- Find $f(0)$ and $f(6)$.
- Find $f(2)$ and $f(-2)$.
- Is $f(3)$ positive or negative?
- Is $f(-1)$ positive or negative?
- For what values of x is $f(x) = 0$?
- For what values of x is $f(x) < 0$?
- What is the domain of f ?
- What is the range of f ?
- What are the x -intercepts?
- What is the y -intercept?
- How many times does the line $y = -1$ intersect the graph?
- How many times does the line $x = 1$ intersect the graph?
- For what value of x does $f(x) = 3$?
- For what value of x does $f(x) = -2$?
- What are the zeros of f ?

In Problems 11–22, determine whether the graph is that of a function by using the vertical-line test. If it is, use the graph to find:

- (a) The domain and range (b) The intercepts, if any (c) Any symmetry with respect to the x -axis, the y -axis, or the origin



In Problems 23–28, answer the questions about the given function.

23. $f(x) = 2x^2 - x - 1$

- Is the point $(-1, 2)$ on the graph of f ?
- If $x = -2$, what is $f(x)$? What point is on the graph of f ?
- If $f(x) = -1$, what is x ? What point(s) is (are) on the graph of f ?
- What is the domain of f ?
- List the x -intercepts, if any, of the graph of f .
- List the y -intercept, if there is one, of the graph of f .
- What are the zeros of f ?

24. $f(x) = -3x^2 + 5x$

- Is the point $(-1, 2)$ on the graph of f ?
- If $x = -2$, what is $f(x)$? What point is on the graph of f ?
- If $f(x) = -2$, what is x ? What point(s) is (are) on the graph of f ?
- What is the domain of f ?
- List the x -intercepts, if any, of the graph of f .
- List the y -intercept, if there is one, of the graph of f .
- What are the zeros of f ?

25. $f(x) = \frac{x+2}{x-6}$

- Is the point $(3, 14)$ on the graph of f ?
- If $x = 4$, what is $f(x)$? What point is on the graph of f ?
- If $f(x) = 2$, what is x ? What point(s) is (are) on the graph of f ?
- What is the domain of f ?
- List the x -intercepts, if any, of the graph of f .
- List the y -intercept, if there is one, of the graph of f .
- What are the zeros of f ?

26. $f(x) = \frac{x^2 + 2}{x + 4}$

- Is the point $(1, \frac{3}{5})$ on the graph of f ?
- If $x = 0$, what is $f(x)$? What point is on the graph of f ?
- If $f(x) = \frac{1}{2}$, what is x ? What point(s) is (are) on the graph of f ?
- What is the domain of f ?
- List the x -intercepts, if any, of the graph of f .
- List the y -intercept, if there is one, of the graph of f .
- What are the zeros of f ?

27. $f(x) = \frac{2x^2}{x^4 + 1}$

- Is the point $(-1, 1)$ on the graph of f ?
- If $x = 2$, what is $f(x)$? What point is on the graph of f ?
- If $f(x) = 1$, what is x ? What point(s) is (are) on the graph of f ?
- What is the domain of f ?
- List the x -intercepts, if any, of the graph of f .
- List the y -intercept, if there is one, of the graph of f .
- What are the zeros of f ?

28. $f(x) = \frac{2x}{x-2}$

- Is the point $(\frac{1}{2}, -\frac{2}{3})$ on the graph of f ?
- If $x = 4$, what is $f(x)$? What point is on the graph of f ?
- If $f(x) = 1$, what is x ? What point(s) is (are) on the graph of f ?
- What is the domain of f ?
- List the x -intercepts, if any, of the graph of f .
- List the y -intercept, if there is one, of the graph of f .
- What are the zeros of f ?

where x is the ground speed (airspeed \pm wind).

- (a) What is the cost when the ground speed is 480 miles per hour; 600 miles per hour?
- (b) Find the domain of C .
- (c) Use a graphing utility to graph the function $C = C(x)$.
- (d) Create a TABLE with TblStart = 0 and $\Delta Tbl = 50$.
- (e) To the nearest 50 miles per hour, what ground speed minimizes the cost per passenger?

34. **Effect of Elevation on Weight** If an object weighs m pounds at sea level, then its weight W (in pounds) at a height of h miles above sea level is given approximately by

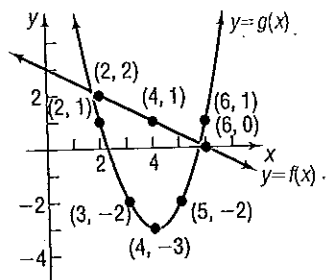
$$W(h) = m \left(\frac{4000}{4000 + h} \right)^2$$

- (a) If Amy weighs 120 pounds at sea level, how much will she weigh on Pike's Peak, which is 14,110 feet above sea level?

- (b) Use a graphing utility to graph the function $W = W(h)$. Use $m = 120$ pounds.

- (c) Create a TABLE with TblStart = 0 and $\Delta Tbl = 0.5$ to see how the weight W varies as h changes from 0 to 5 miles.
- (d) At what height will Amy weigh 119.95 pounds?
- (e) Does your answer to part (d) seem reasonable? Explain.

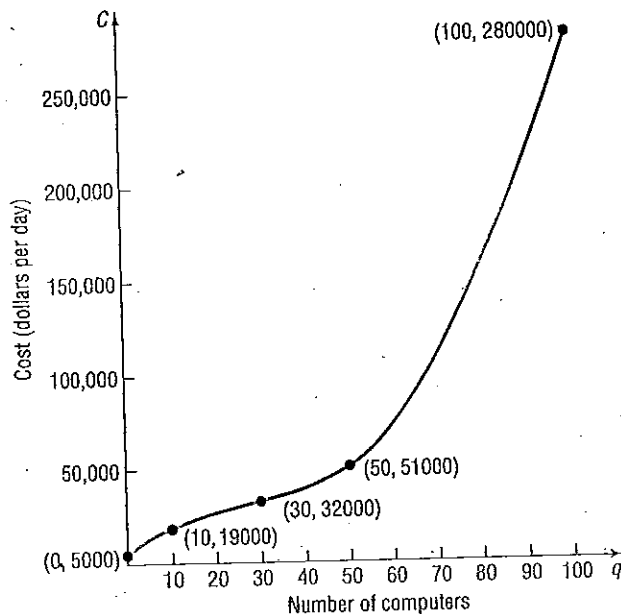
35. The graph of two functions, f and g , is illustrated. Use the graph to answer parts (a) – (f).



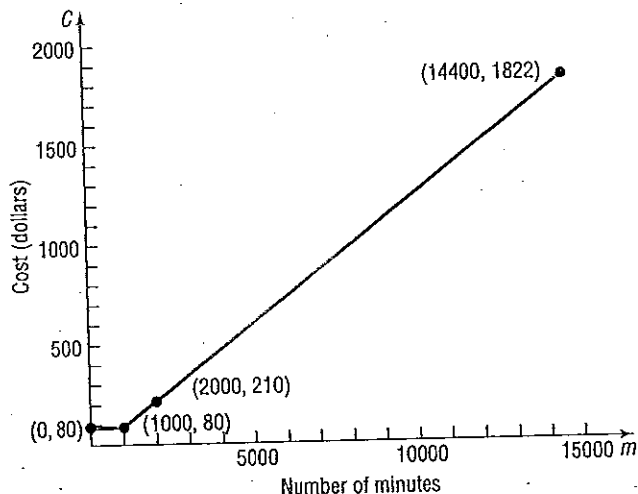
- (a) $(f + g)(2)$
- (b) $(f + g)(4)$
- (c) $(f - g)(6)$
- (d) $(g - f)(6)$
- (e) $(f \cdot g)(2)$
- (f) $\left(\frac{f}{g}\right)(4)$

36. **Reading and Interpreting Graphs** Let C be the function whose graph is given in the next column. This graph represents the cost C of manufacturing q computers in a day.

- (a) Determine $C(0)$. Interpret this value.
- (b) Determine $C(10)$. Interpret this value.
- (c) Determine $C(50)$. Interpret this value.
- (d) What is the domain of C ? What does this domain imply in terms of daily production?
- (e) Describe the shape of the graph.
- (f) The point $(30, 32000)$ is called an *inflection point*. Describe the behavior of the graph around the inflection point.



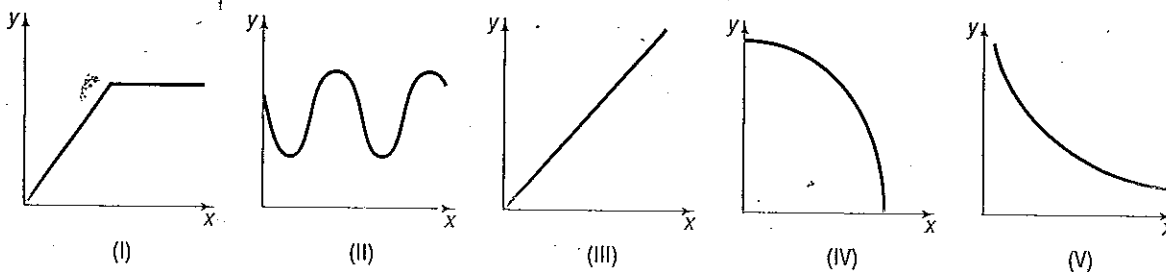
37. **Reading and Interpreting Graphs** Let C be the function whose graph is given below. This graph represents the cost C of using m anytime cell phone minutes in a month for a five-person family plan.



- (a) Determine $C(0)$. Interpret this value.
- (b) Determine $C(1000)$. Interpret this value.
- (c) Determine $C(2000)$. Interpret this value.
- (d) What is the domain of C ? What does this domain imply in terms of the number of anytime minutes?
- (e) Describe the shape of the graph.

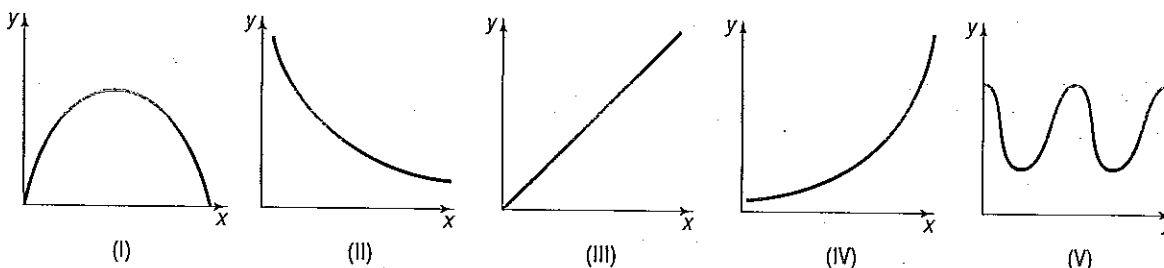
Discussion and Writing

- 38. Describe how you would proceed to find the domain and range of a function if you were given its graph. How would your strategy change if you were given the equation defining the function instead of its graph?
- 39. How many x -intercepts can the graph of a function have? How many y -intercepts can the graph of a function have?
- 40. Is a graph that consists of a single point the graph of a function? Can you write the equation of such a function?
- 41. Match each of the following functions with the graph on the next page that best describes the situation.
 - (a) The cost of building a house as a function of its square footage
 - (b) The height of an egg dropped from a 300-foot building as a function of time
 - (c) The height of a human as a function of time
 - (d) The demand for Big Macs as a function of price
 - (e) The height of a child on a swing as a function of time?



42. Match each of the following functions with the graph that best describes the situation.

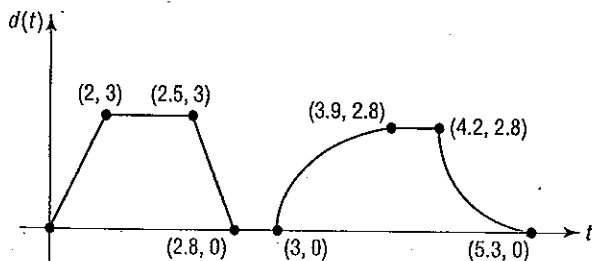
- The temperature of a bowl of soup as a function of time
- The number of hours of daylight per day over a 2-year period
- The population of Texas as a function of time
- The distance traveled by a car going at a constant velocity as a function of time
- The height of a golf ball hit with a 7-iron as a function of time



43. Consider the following scenario: Barbara decides to take a walk. She leaves home, walks 2 blocks in 5 minutes at a constant speed, and realizes that she forgot to lock the door. So Barbara runs home in 1 minute. While at her doorstep, it takes her 1 minute to find her keys and lock the door. Barbara walks 5 blocks in 15 minutes and then decides to jog home. It takes her 7 minutes to get home. Draw a graph of Barbara's distance from home (in blocks) as a function of time.

44. Consider the following scenario: Jayne enjoys riding her bicycle through the woods. At the forest preserve, she gets on her bicycle and rides up a 2000-foot incline in 10 minutes. She then travels down the incline in 3 minutes. The next 5000 feet is level terrain and she covers the distance in 20 minutes. She rests for 15 minutes. Jayne then travels 10,000 feet in 30 minutes. Draw a graph of Jayne's distance traveled (in feet) as a function of time.

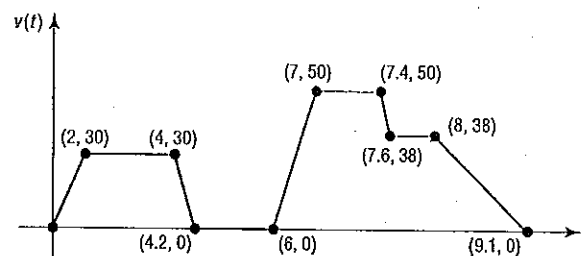
45. The following sketch represents the distance d (in miles) that Kevin was from home as a function of time t (in hours). Answer the questions by referring to the graph. In parts (a) – (g), how many hours elapsed and how far was Kevin from home during this time?



- From $t = 0$ to $t = 2$
- From $t = 2$ to $t = 2.5$
- From $t = 2.5$ to $t = 2.8$

- From $t = 2.8$ to $t = 3$
- From $t = 3$ to $t = 3.9$
- From $t = 3.9$ to $t = 4.2$
- From $t = 4.2$ to $t = 5.3$
- What is the farthest distance that Kevin was from home?
- How many times did Kevin return home?

46. The following sketch represents the speed v (in miles per hour) of Michael's car as a function of time t (in minutes).



- Over what interval of time was Michael traveling fastest?
- Over what interval(s) of time was Michael's speed zero?
- What was Michael's speed between 0 and 2 minutes?
- What was Michael's speed between 4.2 and 6 minutes?
- What was Michael's speed between 7 and 7.4 minutes?
- When was Michael's speed constant?

47. Draw the graph of a function whose domain is $\{x \mid -3 \leq x \leq 8, x \neq 5\}$ and whose range is $\{y \mid -1 \leq y \leq 2, y \neq 0\}$. What point(s) in the rectangle $-3 \leq x \leq 8, -1 \leq y \leq 2$ cannot be on the graph? Compare your graph with those of other students. What differences do you see?

48. Is there a function whose graph is symmetric with respect to the x -axis? Explain.

49. Explain why the vertical-line test works.

'Are You Prepared?' Answers

1. $(-4, 0)$, $(4, 0)$, $(0, -2)$, $(0, 2)$

2. False