

1.5 Transformations

Transformations of functions: horizontal + vertical shifts, dilations, reflections

vertical shift

$f(x) + a$: shift up by a units
ex) $f(x) = x^2 + 2$
 $g(x) = |x| + 4$

$f(x) - a$: shifts down by a units

horizontal shift

$f(x + a)$: horizontal shift to the left by a units

ex) $f(x) = (x + 2)^2$
parent fn is x^2

$f(x) = |x|$, $g(x) = |x + 2|$

$h(x) = \sqrt{x}$, $m(x) = \sqrt{x + 2}$

$f(x - a)$: horiz. shift to the right by a units

ex) $f(x) = (x - 2)^2$

5/x

reflections

$f(-x)$: reflection over y-axis

ex) $f(x) = \sqrt{x}$, $g(x) = \sqrt{-x}$

$-f(x)$: reflection over x-axis

ex) $f(x) = \sqrt{x}$; $g(x) = -\sqrt{x}$

$af(x)$: vertical dilation

$a > 1$ stretch (thinner)

$1 > a > 0$ compression (wider)

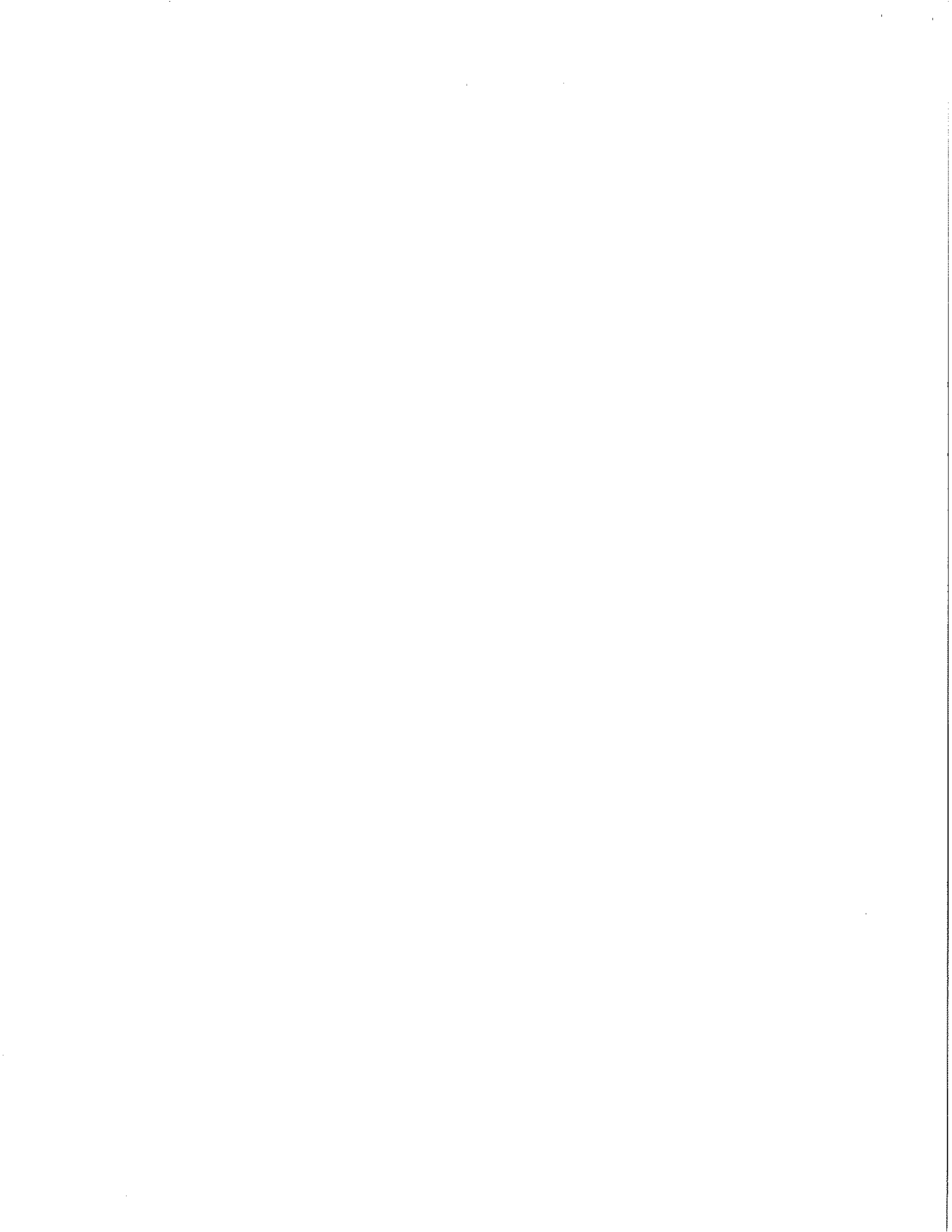
ex) $f(x) = x^3$, $g(x) = 2x^3$ [$2f(x)$]

$f(ax)$: horizontal dilation

$a > 1$ compression (thinner)

$1 > a > 0$ stretch (wider)

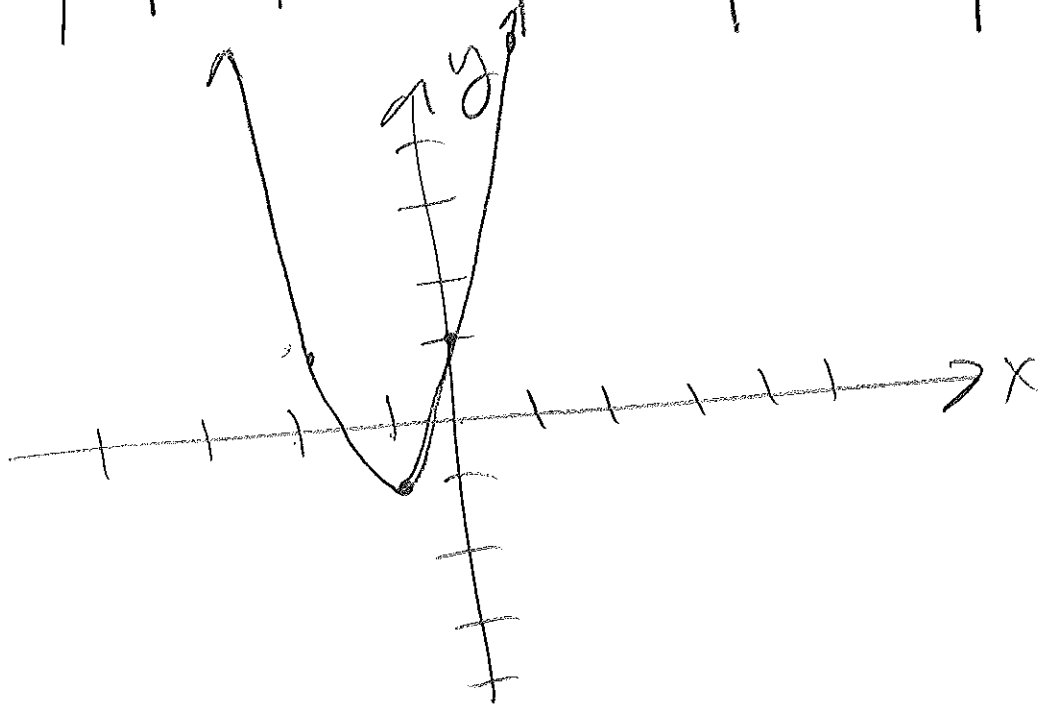
ex) $f(x) = x^3$, $g(x) = (2x)^3$ [$f(2x)$]



Graph Transformed f(x)s

$$f(x) = x^2 \Rightarrow g(x) = 2(x+1)^2 - 1$$

x	-2	-1	0	1
g(x)	$2(-2+1)^2 - 1$ $2(-1)^2 - 1$ 1	$2(-1+1)^2 - 1$ -1	$2(0+1)^2 - 1$ 1	$2(1+1)^2 - 1$ 7
y	1	-1	1	7



Rules

		point on graph before transf.	after transf.
Left Shift	$f(x+a)$	(x, y)	$\rightarrow (x-a, y)$
Right Shift	$f(x-a)$	(x, y)	$\rightarrow (x+a, y)$
horiz compression	$f(ax)$	(x, y)	$\rightarrow (\frac{1}{a}x, y)$
vertical dilation	$a f(x)$	(x, y)	$\rightarrow (x, ay)$
horiz stretch	$f(\frac{1}{a}x)$	(x, y)	$\rightarrow (ax, y)$
reflection y-axis	$f(-x)$	(x, y)	$\rightarrow (-x, y)$
reflection x-axis	$-f(x)$	(x, y)	$\rightarrow (x, -y)$

ex) $(2, -4) \xrightarrow[\text{x-axis}]{\text{reflection}}$ $(2, 4)$

$(3, 1) \xrightarrow[\text{of } 2]{\text{v. dilation}}$ $(3, 2)$

