2-sample z interval

10.1 Comparing Two Proportions

Learning Objectives:

1. Describe the shape, center, and spread of the sampling distributions of $\hat{p}_1 - \hat{p}_2$.
2. Determine whether the conditions are met for doing inference about $\hat{p}_1 - \hat{p}_2$.
3. Construct and interpret a confidence interval to compare two proportions.
4. Perform a significance test to compare two proportions.

Vocabulary: two sample z interval for $p_1 - p_2$, pooled (combined) sample proportion, randomization distribution.

*Read 612–615 **Read Case Study on p. 609****

What is meant by "the sampling distribution of the difference between two proportions"?

All possible values of $\hat{p}_1 - \hat{p}_2$ & how often they occur

$\hat{p}_1 - \hat{p}_2$ = proportion of successes from independent random samples.

What are the shape, center, and spread of the sampling distribution of $\hat{p}_1 - \hat{p}_2$? (see box on p. 614)

Shape: Approx. Normal if: $n_1p_1,\ n_1(1-p_1),\ n_2p_2,\ n_2(1-p_2)$ all ≥ 10

Center: Mean: $\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2$

Spread: S.d.: $\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$

Only if sampling is "good"
- random
- 10% condition

<u>Alternate Example</u>: Nathan and Kyle both work for the Department of Motor Vehicles, but they live in different states. In Nathan's state, 80% of the registered cars are made by American manufacturers. In Kyle's state, only 60% of the registered cars are made by American manufacturers. Nathan selects a random sample of 100 cars in his state and Kyle selects a random sample of 70 cars in his state. Let $\hat{p}_N - \hat{p}_K$ be the difference in the sample proportion of cars made by American manufacturers.

$p_N = .8$, $n_N = 100$, $p_K = .6$, $n_K = 70$

(a) What is the shape of the sampling distribution of $\hat{p}_N - \hat{p}_K$? Why?

$n_N p_N$: $100(.8) = 80 \geq 10$ & $n_K p_K$: $70(.6) = 42 \geq 10$

$n_N(1-p_N)$: $100(.2) = 20 \geq 10$ & $n_K(1-p_K)$: $70(.4) = 28 \geq 10$

So, samp. distrib of $\hat{p}_N - \hat{p}_K$ is ~Normal

(b) Find the mean of the sampling distribution. Show your work.

$\mu_{\hat{p}_N-\hat{p}_K} = p_N - p_K = .8 - .6 = .2$

(c) Find the standard deviation of the sampling distribution. Show your work.

Since 100 < 10% cars in Nathan's state & 70 < 10% cars in Kyle's state, we can calculate $\sigma_{\hat{p}_N-\hat{p}_K} = \sqrt{\dfrac{.8(.2)}{100} + \dfrac{.6(.4)}{70}} = .0709$

Read 616–619

What are the conditions for calculating a $\boxed{\text{two-sample } z \text{ interval for } p_1 - p_2}$?

- Random - MUST check BOTH samples / experiments
- Independent / 10% condition - for both
- Large counts: $n_1 p_1, n_1(1-p_1), n_2 p_2, n_2(1-p_2)$ all ≥ 10

$\rightarrow \sigma_{\hat{p}_1 - \hat{p}_2}$ use p not $\hat{p}$

What is the standard error of $\hat{p}_1 - \hat{p}_2$? How is this different than the standard deviation of $\hat{p}_1 - \hat{p}_2$? What does this measure?

measures how far sample proportion difference will typically vary from pop. difference in repeated samples

$$SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

What is the formula for a two-sample z interval for $p_1 - p_2$? Is this on the formula sheet?

$$CI_{\hat{p}_1 - \hat{p}_2} = \underbrace{(\hat{p}_1 - \hat{p}_2)}_{\text{pnt estimate/ Statistic}} \pm \underbrace{z^*}_{z \text{ critical}} \underbrace{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}_{SE}$$

Alternate Example: *Gun Control*

Have opinions changed about gun control? Gallup regularly asks random samples of U.S. adults their opinion on a variety of issues. In a poll of 1011 U.S. adults in January 2013, 38% responded that they "were dissatisfied with the nation's gun laws and policies, and want them to be stricter." In a similar poll of 1011 adults in January 2012, only 25% agreed with this statement.

$\hat{p}_{13} = 0.38$ $\quad$ $\hat{p}_{12} = 0.25$

$n_{13} = 1011$ $\quad$ $n_{12} = 1011$

(a) Explain why we should use a confidence interval to estimate the change in opinion rather than just saying that the percentage increased by 13 percentage points.

b/c of sampling variability, the difference of .13 is unlikely to be exactly correct.

$\hat{p}_{13} = .38$ $\hat{p}_{12} = .25$

$n_{13} = 1011$ $n_{12} = 1011$

(b) Use the results of these polls to construct and interpret a 90% confidence interval for the change in the proportion of U.S. adults who would agree with the statement about gun laws.

State: We want to estimate $p_{13} - p_{12}$ at 90% confidence level where

p_{13} = true proportion adults dissatisfied w/ gun laws in 2013

p_{12} = " " " " " " 2012.

Plan: Construct $\boxed{\text{2-sample z interval}}$ if conditions are met

- Both samples are random
- Both samples are independent: 1011 < 10% of all adults in US in 2012 & 2013
- $\underset{n_{13}}{1011}(\underset{\hat{p}_{13}}{.38}) = 384.10$, $\underset{n_{13}}{1011}(\underset{1-\hat{p}_{13}}{.62}) = 626.82$, $\underset{n_{12}}{1011}(\underset{\hat{p}_{12}}{.25}) = 252.75$, $\underset{n_{12}}{1011}(\underset{1-\hat{p}_{12}}{.75}) = 758.25$

  all ≥ 10 $\therefore$ approx. Normal

Do: $(\underset{\hat{p}_{13}}{.38} - \underset{\hat{p}_{12}}{.25}) \pm \underset{\text{invNorm}(.05)}{1.645}\sqrt{\frac{.38(.62)}{1011} + \frac{.25(.75)}{1011}} = .13 \pm 0.034 = (.096, .164)$

Conclude: We are 90% confident that the true prop change in adult opinion of gun laws from 2012 to 2013 is captured in the interval (.096, .164).

(c) Based on the interval, is there convincing evidence that opinions about gun control have changed?

Yes! Since 0 (ie. no change) is NOT in the interval, there is convincing evidence of change.

Can you use your calculator for the Do step? Are there any drawbacks?

*use integers only!! (round)

STAT → TESTS B: 2-Prop Z Int

x_1 = # of successes in sample 1 $(.38 \cdot 1011 = 384.18)$

x_2 = " " " " " 2 $(.25 \cdot 1011 = 252.75)$ round

⇒ (.04592, .16323)

HW page 629 (1–11 odd)

Significance Tests for a Difference in Proportions

Example: *Hungry Children*

Researchers designed a survey to compare the proportions of children who come to school without eating breakfast in two low-income elementary schools. An SRS of 80 students from School 1 found that 19 had not eaten breakfast. At School 2, an SRS of 150 students included 26 who had not had breakfast. More than 1500 students attend each school.

$\hat{p}_1 = 19/80 = 0.24$ $\quad \hat{p}_2 = \frac{26}{150} = 0.17$

$n_1 = 80$ $\quad n_2 = 150$

What are the two explanations for why the SRS from each school do not have the same result?

① Proportion "No breakfast" at each school is the same & we got different sample $\hat{p}$'s by chance ($H_0: p_1 = p_2 \rightarrow p_1 - p_2 = 0$)

② Proportion "No breakfast" at each school really are different ($H_a: p_1 \neq p_2 \rightarrow p_1 - p_2 \neq 0$)

Read 619-624

What are the conditions for conducting a two-sample *z* test for a difference in proportions?

* Same as CI for $\hat{p}_1 - \hat{p}_2$

* Be careful not to accept H_0!

- Random & independent samples
  - ↳ 10% condition
- Large Counts ($n\hat{p}, n(1-\hat{p}) \geq 10$) for both!

We can get a more precise estimate of the standard deviation of a sample statistic by combining, called the pooled ($\hat{p}_c$) or combined sample proportion of the estimates given by two independent samples.

We always assume H_0 is true so $H_0: p_1 = p_2$. We can combine the sample proportion for a more precise estimate of true p.

Hungry Children Two-way table

	School		**Total**
Breakfast?	1	2	
No	19	26	**45**
Yes	61	124	**185**
Total	**80**	**150**	**230**

$$\boxed{\hat{p}_C = \frac{X_1 + X_2}{n_1 + n_2}}$$

$$\hat{p}_C = \frac{19+26}{80+150} = \frac{45}{230} = 0.196$$

What standard error do we use for a two sample *z* test for a difference in proportions?

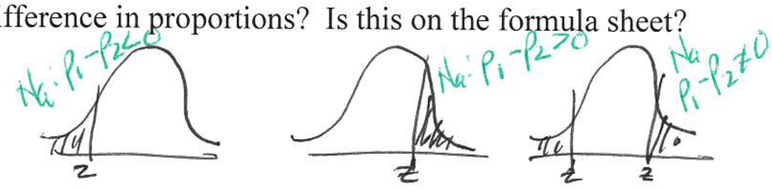
$$SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_C(1-\hat{p}_C)}{n_1} + \frac{\hat{p}_C(1-\hat{p}_C)}{n_2}}$$

* uses pooled proportion!!

What is the test statistic for a two-sample *z* test for a difference in proportions? Is this on the formula sheet?

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\frac{\hat{p}_C(1-\hat{p}_C)}{n_1} + \frac{\hat{p}_C(1-\hat{p}_C)}{n_2}}}$$

→ H_0 is always $p_1 - p_2 = 0$ or $p_1 = p_2$ unless otherwise stated



p_1 : 2005-06 $\quad p_2$: 1988-94

$\hat{p}_1 = .195 \quad \hat{p}_2 = 0.15$

$n_1 = 1800 \quad n_2 = 3000$

Alternate Example: *Hearing loss*

Are teenagers going deaf? In a study of 3000 randomly selected teenagers in 1988–1994, 15% showed some hearing loss. In a similar study of 1800 teenagers in 2005–2006, 19.5% showed some hearing loss. (These data are reported in *Arizona Daily Star*, August 18, 2010).

key word for "test"

(a) Do these data give convincing evidence that the proportion of all teens with hearing loss has increased?

State: Test $H_0: p_1 - p_2 = 0$ vs. $H_a: p_1 - p_2 > 0$ at $\alpha = 0.05$

where p_1 = proportion of all teens w/ hearing loss in 2005-06

p_2 = " " " " " " 1988-94

Plan: Perform $\boxed{\text{2-sample z test}}$ if conditions are met:

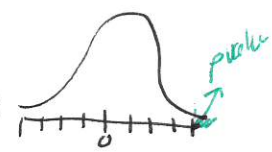
- two independent & random samples taken
- More than 10(1800) = 18,000 teens in 2005-06
  10(3000) = 30,000 teens in 1988-94
- $\overset{n_1}{1800}(\overset{\hat{p}_1}{.195}) = 351$, $\overset{n_1}{1800}(\overset{1-\hat{p}_1}{.805}) = 1449$, $\overset{n_2}{3000}(\overset{\hat{p}_2}{.15}) = 450$, $\overset{n_2}{3000}(\overset{1-\hat{p}_2}{.85}) = 2550$
  all ≥ 10

Do:

	Hearing Loss	None	
88-94	450	2550	3000
05-06	351	1449	1800
	801		4800

$$\hat{p}_c = \frac{450 + 351}{3000 + 1800} = \frac{801}{4800} = 0.167$$

$$z = \frac{(.195 - .15) - 0}{\sqrt{\frac{.167(1-.167)}{1800} + \frac{.167(1-.167)}{3000}}} = \boxed{4.05}$$



normalcdf(4.05, 10000)

$\boxed{\text{p-value} \approx 0}$

Conclude: Since p-value ≈ 0 < 0.05 (α), we reject H_0. We have convincing evidence that the proportion of all teens w/ hearing loss has increased from 1988-94 to 2005-06.

(b) Between the two studies, Apple introduced the iPod. If the results of the test are statistically significant, can we blame iPods for the increased hearing loss in teenagers?

No! Can't establish cause & effect relationship w/out an experiment w/ randomly assigned treatments, etc.

It's possible that other variables (car stereos are too loud) are causing the hearing loss

Is it OK to use your calculator for the Do step? Are there any drawbacks?

YES! Please use calculator!!

STAT → TESTS → 6: 2-Prop Z Test

* X_1 & X_2 are integers, NOT proportions

* Don't use 2-sample Z test!!

Given:

z

p (p-value)

$\left.\begin{matrix}\hat{p}_1 \\ \hat{p}_2\end{matrix}\right\}$ use this to check your inputs

$\hat{p}$ = pooled proportion!

HW page 631 (13–19 odd)

Inference for Experiments

Read 625–627

What mistake do students often make when defining parameters in experiments? How can you avoid it?

* It is better to use present or future tense.
* Use language that refers to the sample
  "proportion who took..."
  "proportion of people in ___ group"

Review in table p. 267

Example: *Yawning Seed*

The Mythbusters team conducted an experiment involving 50 subjects. Each subject was placed in a booth for an extended period of tiem and monintored by hidden camera. Thirty four subjects were randomly assigned to be given a "yawn seed" by on of the experimenters; that is, the experimenter yawned in the subject's presence before leaving the room. The remaining 16 subject were given no yawn seed. Define the parameters the researchers are comparing.

p_1 = true proportion of people like these who would yawn when given a "seed"

p_2 = true proportion of people like these who would NOT yawn...

Alternate Example: *Cash for quitters*

In an effort to reduce health care costs, General Motors sponsored a study to help employees stop smoking. In the study, half of the subjects were randomly assigned to receive up to $750 for quitting smoking for a year while the other half were simply encouraged to use traditional methods to stop smoking. None of the 878 volunteers knew that there was a financial incentive when they signed up. At the end of one year, 15% of those in the financial rewards group had quit smoking while only 5% in the traditional group had quit smoking. Do the results of this study give convincing evidence that a financial incentive helps people quit smoking compared to traditional methods? (These data are reported in *Arizona Daily Star,* February 11, 2009)

$\hat{p}_1 = .15,\ n_1 = 439;\ \hat{p}_2 = .05,\ n_2 = 439$

State: Test $H_0: p_1 - p_2 = 0$ vs. $H_a: p_1 - p_2 > 0$ @ $\alpha = 0.05$
$(p_1 = p_2)$ $(p_1 > p_2)$

where p_1 = true quitting rate for employees like these w/ "incentives"
p_2 = " " " " " " w/o "

Plan: Perform a 2-sample z test if conditions are met
- Treatments were randomly assigned
- 10%: not needed for experiments!
- $439(.15) = 66,\ 439(.85) = 373,\ 439(.05) = 22,\ 439(.95) = 417$
  all ≥ 10

Do: Using calculater: 2-PropZTest
$z = 4.945$, p-value ≈ 0

Conclude: Since p-value $\approx 0 < \alpha = .05$, we reject H_0. There is convincing evidence that financial incentives help employees like these quit smoking.

HW page 630 (21, 23, 25–30)