2.2 Density curves and Normal Distributions

Learning Targets

1. Estimate the relative location of the median and mean on a density curve.
2. Use the 68-95-99.7 rule to estimate areas (proportions of values) in a Normal distribution.
3. Use Table A or technology to find (i) the proportion of z-values in a specified interval, or (ii) a z-score from a percentile in the standard Normal distribution.
4. Use Table A or technology to find (i) the proportion of values in a specified interval, or (ii) the value that corresponds to a given percentile in any Normal distribution.
5. Determine whether a distribution of data is approximately Normal from graphical and numerical evidence.

Vocabulary: density curve, Normal curves, Normal distributions, 68-95-99.7 rule

Read 103–107

What is a density curve? When would we use a density curve? Why?

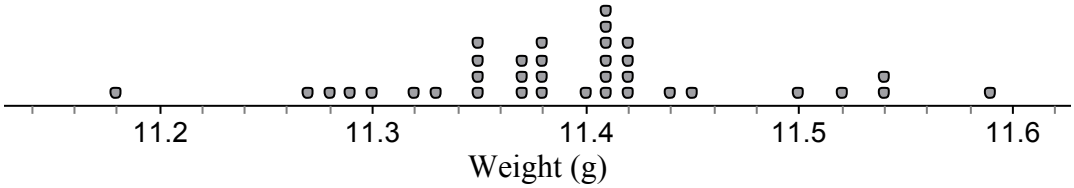
How can you identify the mean and median of a density curve?

Read 108–109

According to the CDC, the heights of 12-year-old males are approximately Normally distributed with a mean of 149 cm and a standard deviation of 9 cm. Sketch this distribution, labeling the mean and the points one, two, and three standard deviations from the mean.

***Activity*:** For each of the approximately Normal distributions below, calculate the percentage of values within one standard deviation of the mean, within two standard deviations of the mean, and within three standard deviations of the mean.

1. Here is a dotplot showing the weights (in grams) of 36 Oreo cookies. The mean of this distribution is 11.392 g and the standard deviation is 0.081 g.

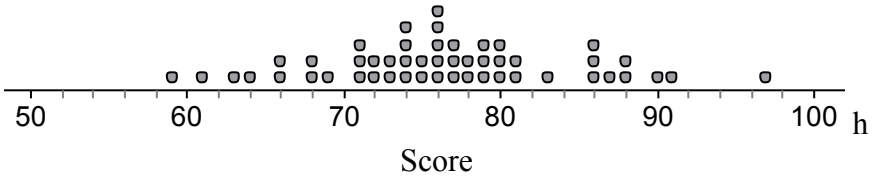


Mean ± 1 SD: _____ to _____ % within 1 SD: _____

Mean ± 2 SD: _____ to _____ % within 2 SD: _____

Mean ± 3 SD: _____ to _____ % within 3 SD: _____

2. Here is dotplot showing the scores for 50 students on an algebra test. The mean of this distribution is 76.4 and the standard deviation is 7.9.

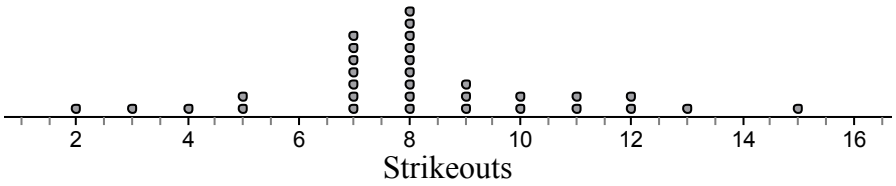


Mean ± 1 SD: _____ to _____ % within 1 SD: _____

Mean ± 2 SD: _____ to _____ % within 2 SD: _____

Mean ± 3 SD: _____ to _____ % within 3 SD: _____

3. Here is a dotplot of Tim Lincecum's strikeout totals for each of the 32 games he pitched in during the 2009 regular season. The mean of this distribution is 8.2 with a standard deviation of 2.8.



Mean ± 1 SD: _____ to _____ % within 1 SD: _____

Mean ± 2 SD: _____ to _____ % within 2 SD: _____

Mean ± 3 SD: _____ to _____ % within 3 SD: _____

4. All three of the distributions above were approximately Normal in shape. Based on these examples, about what percent of the observations would you expect to find within one standard deviation of the mean in a Normal distribution?

Within two standard deviations of the mean?

Within three standard deviations of the mean?

Read 109–112

What is the 68-95-99.7 rule? When does it apply?

Do you need to know about Chebyshev's inequality?

Using the earlier example, about what percentage of 12-year-old boys will be over 158 cm tall?

About what percentage of 12-year-old boys will be between 131 and 140 cm tall?

Suppose that a distribution of test scores is approximately Normal and the middle 95% of scores are between 72 and 84. What are the mean and standard of this distribution?

Can you calculate the percent of scores that are above 80? Explain.

HW: page 102 (25–30), page 128 (33-45 odd)

2.2 Normal Calculations

Read 112–114

What is the standard Normal distribution?

Find the proportion of observations from the standard Normal distribution that are:

(a) less than 0.54

(b) greater than –1.12

(c) greater than 3.89

(d) between 0.49 and 1.82.

(e) within 1.5 standard deviations of the mean

A distribution of test scores is approximately Normal and Joe scores in the 85^{th} percentile. How many standard deviations above the mean did he score?

In a Normal distribution, Q_1 is how many SD below the mean?

Alternate Example: Serving Speed

In the 2008 Wimbledon tennis tournament, Rafael Nadal averaged 115 miles per hour (mph) on his first serves. Assume that the distribution of his first serve speeds is Normal with a mean of 115 mph and a standard deviation of 6 mph.

(a) About what proportion of his first serves would you expect to be slower than 103 mph?

(b) About what proportion of his first serves would you expect to exceed 120 mph?

(c) What percent of Rafael Nadal's first serves are between 100 and 110 mph?

(d) The fastest 30% of Nadal's first serves go at least what speed?

(e) A different player has a standard deviation of 8 mph on his first serves and 20% of his serves go less than 100 mph. If the distribution of his serve speeds is approximately Normal, what is his average first serve speed?

HW: page 129 (47–57 odd)

2.2: Using the Calculator for Normal Calculations

How do you do Normal calculations on the calculator? What do you need to show on the AP exam?

Suppose that Clayton Kershaw of the Los Angeles Dodgers throws his fastball with a mean velocity of 94 miles per hour (mph) and a standard deviation of 2 mph and that the distribution of his fastball speeds can be modeled by a Normal distribution.

(a) About what proportion of his fastballs will travel less than 100 mph?

(b) About what proportion of his fastballs will travel greater than 100 mph?

(c) About what proportion of his fastballs will travel less than 90 mph?

(d) About what proportion of his fastballs will travel between 93 and 95 mph?

(e) What is the 30^{th} percentile of Kershaw's distribution of fastball velocities?

(f) What fastball velocities would be considered low outliers for Kershaw?

(g) Suppose that a different pitcher's fastballs have a mean velocity of 92 mph and 40% of his fastballs go less than 90 mph. What is his standard deviation of his fastball velocities, assuming his distribution of velocities can be modeled by a Normal distribution?

HW page 130 #54, 55, 57, 59, 68–73

2.2 Assessing Normality

Read 121–122

The measurements listed below describe the useable capacity (in cubic feet) of a sample of 36 side-by-side refrigerators. (Source: *Consumer Reports*, May 2010) Are the data close to Normal?

12.9	13.7	14.1	14.2	14.5	14.5	14.6	14.7	15.1	15.2	15.3	15.3
15.3	15.3	15.5	15.6	15.6	15.8	16.0	16.0	16.2	16.2	16.3	16.4
16.5	16.6	16.6	16.6	16.8	17.0	17.0	17.2	17.4	17.4	17.9	18.4

Read 122–125

When looking at a Normal probability plot, how can we determine if a distribution is approximately Normal?

Sketch a Normal probability plot for a distribution that is strongly skewed to the left.

HW: Do Ch 2 Practice Test