Pre Calc
NOTES

KEY

.W 1/18/17

3.1 Polynomial Functions & Models

Polynomial functions (fxns) has the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0, \quad for\ a_n \neq 0$$

n = non-negative integer

- coefficients, $a_n, a_{n-1}, \ldots, a_1, a_0$, are real numbers
- leading term is $a_n x^n$, and a_n is the lead coefficient
- a_0 is the constant term
- the degree of a polynomial fxn is the largest power of *x* that appears
- standard form of a polynomial fxn is when a polynomial is written in descending order of degree

Graphs of a polynomial function are smooth & continuous, which means there are no corners or cusps and no gaps.

IDENTIFYING POLYNOMIAL FXNS

Determine if the following functions are polynomial functions. If so, identify the degree, write in standard form, identify the leading term & constant term. If not, explain why.

a. $p(x) = 5x^3 - \frac{1}{4}x^2 + 7x - 9$

yes, already in standard form
degree: 3, l.t $= 5x^3$, c.t $= -9$

b. $f(x) = x + 2 - 3x^4$

yes: $f(x) = -3x^4 + x + 2$
degree: 4, l.t: $-3x^4$, c.t: 2

c. $g(x) = \sqrt{x}$

No, $\sqrt{x} = x^{1/2}$ → should be an integer

d. $h(x) = \frac{x^2-2}{x^3-1}$

No, rational

e. $G(x) = 8$

yes, already in standard form
degree: 8 no l.t., c.t = 8

f. $H(x) = -2x^3(x-1)^2$

$-2x^3(x-1)(x-1)$
$-2x^3(x^2-2x+1)$
$-2x^5+4x^4-2x^3$

yes, already in standard form
degree: 5, l.t: $-2x^5$, c.t = 0/none

POWER FUNCTIONS

Power functions are a form of polynomial function that is a monomial.

$$f(x) = ax^n \qquad \text{where } a \text{ is a real number}, \; n > 0 \text{ is an integer}$$

Examples of Power Functions:

$f(x) = 3x$	$f(x) = -5x^2$	$f(x) = 8x^3$	$f(x) = -5x^4$
degree 1 linear	degree 2 quadratic (parabola)	degree 3 cubic fxn (seat fxn)	degree 4

Properties of power functions with **even** degrees:

- symmetric about the y-axis
- always contain the points (0,0), (-1,1), and (1,1).
- domain is $(-\infty,\infty)$.
- as the power increases, the graph near (0, 0) gets flatter
- the graph gets steeper when x < -1 and x > 1

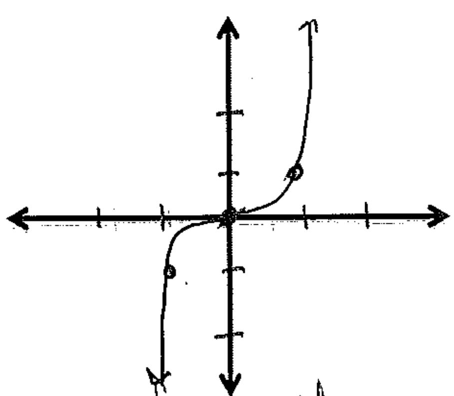
Properties of power functions with **odd** degrees:

- domain is $(-\infty,\infty)$, range is $(-\infty,\infty)$
- symmetric about the origin
- always contain the points (0,0), (-1,-1), and (1,1)
- as the power increases, the graph near (0, 0) gets flatter

GRAPH POLYNOMIAL FXNS USING TRANSFORMATIONS

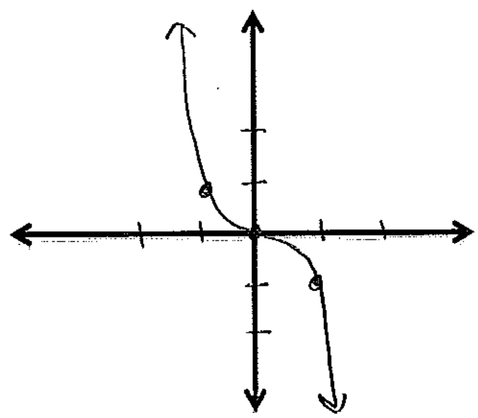
Recall transformations: vertical shifts, horizontal shifts, dilations, reflections.

A. Graph $f(x) = 1 - x^5$



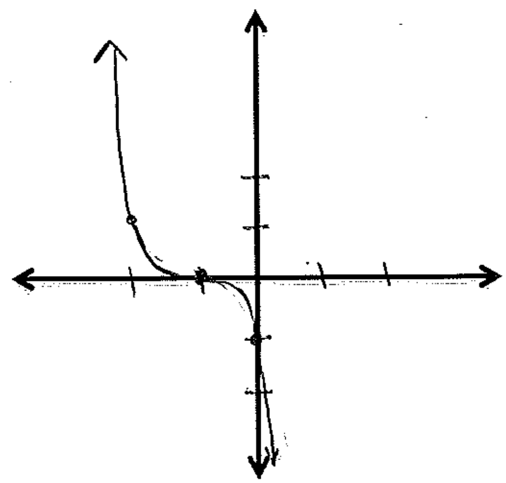
$y = x^5$ odd

has the pnts (0,0) (-1,-1) (1,1)



$y = -x^5$

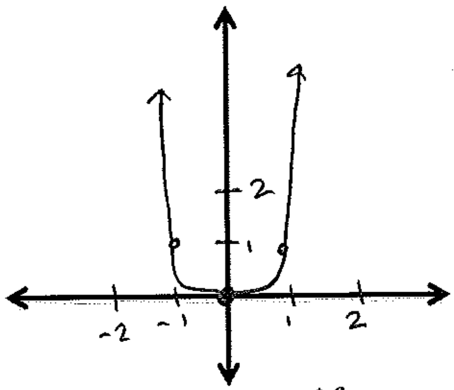
reflection about x-axis



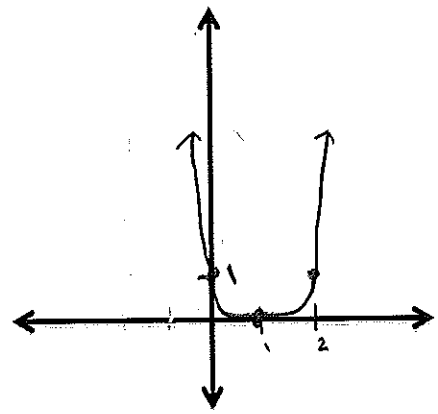
$y = -x^5 + 1$

shifted left by 1

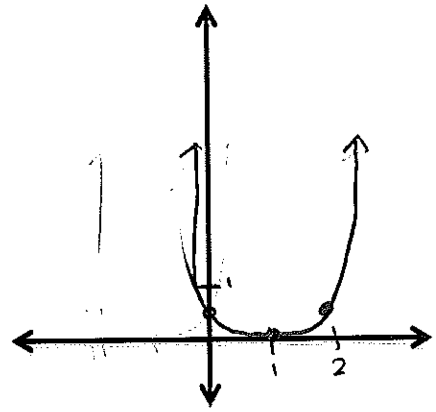
B. Graph $f(x) = \frac{1}{2}(x-1)^4$



$y = x^4$ even
Know the pts
(0,0) (-1,1) (1,1)



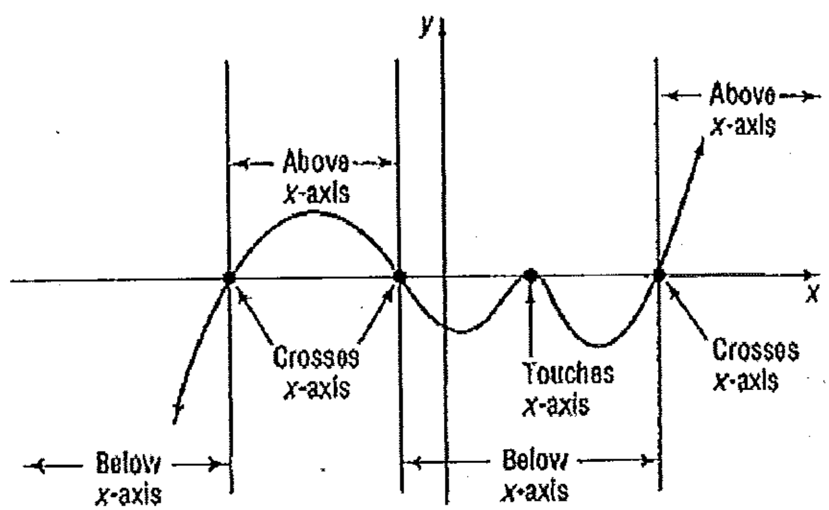
$y = (x-1)^4$
shifted right by 1



$y = \frac{1}{2}(x-1)^4$
V. compression of 1/2

IDENTIFY REAL ZEROS OF A POLYNOMIAL FXN

Intercepts of a polynomial fxn may cross or touch the x-intercept. Whether it crosses or touches is determined by the **multiplicity**. Notice between intercepts (or zeros), the graph is either above or below the x-axis.



When a polynomial is in factored form, it is easy to determine the x-intercepts (or zeros)

$$f(x) = (x-1)^2(x+3)$$

The zeros are x = 1 and x = -3. Therefore, $f(1) = 0$ *and* $f(-3) = 0$. Points on the graph of f are (0,1) & (0,-3).

Therefore, if $f(r) = 0$, then

a. r is called a real zero.
b. r is an x-intercept of the graph of f
c. $x - r$ is a factor of f
d. r is a solution to the equation $f(x) = 0$

FINDING A POLYNOMIAL FXN FROM ITS ZEROS

Find a polynomial fxn of degree 3 whose zeros are -3, 2, and 5

$$f(x) = a(x+3)(x-2)(x-5$$

where a is a non-zero real number that is the dilation factor

MULTIPLICITY & ZEROS

Multiplicity refers to the number of times that its associated factor appears in the polynomial. For example $(x+4)$ has a multiplicity of 1 because the exponent is 1 and $(x-2)^2$ has a multiplicity of 2 because the exponent is 2.

Given: $f(x) = 5x^2(x+2)\left(x - \frac{1}{2}\right)^4$

The zeros are: 0, -2, 1/2

The multiplicity is the exponent on the factor.

Zeros	Multiplicity
0	2
-2	1
1/2	4

Even multiplicity: graph touches the x-axis at the corresponding zero. (like a parabola)
Odd multiplicity: graph crosses the x-axis at the corresponding zero.