KEY

Pre Calc
NOTES

F 1/20/17

3.1 Polynomial Functions & Models Part II

TURNING POINTS

The points at which a graph changes direction are called turning points. Each turning point yields a local maximum or local minimum.

THEOREM

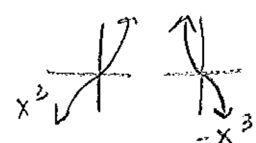
If f is a polynomial function of a degree n, then the graph of f has at most $n - 1$ turning points

If the graph of a polynomial function f has $n - 1$ turning points, then the degree of f is at least n.

Example: Given $f(x) = 2x^6 - 7x^4 + 3x^2 - 1$, the maximum number of turning points is 5.

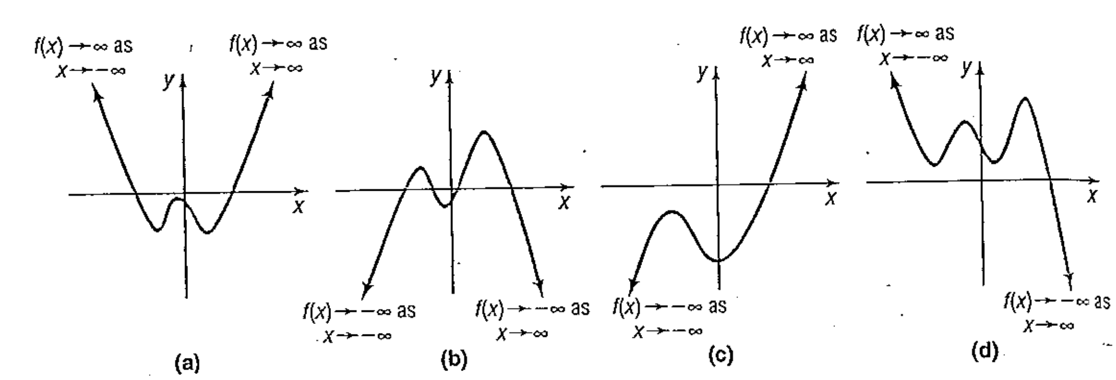
END BEHAVIOR

The behavior of the graph of a fxn for large values of x, either positive or negative, is referred to as its end behavior. The behavior of a polynomial resembles the graph of the power fxn, which can be determined by looking at the leading term.



Example: Given $f(x) = -3x^3 - x^2 + x + 9$, the end behavior resembles $-3x^3$. In other words, when x gets more negative, the graph approaches ∞ and when the x value gets more positive, the graph approaches $-\infty$.

Notation used to describe end behavior is as $x \to -\infty$, $f(x) \to \infty$ & as $x \to \infty$, $f(x) \to -\infty$

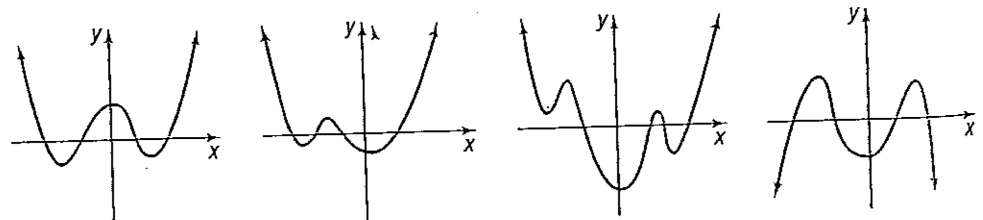


$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$

IDENTIFYING THE GRAPH OF A POLYNOMIAL FXN

Which of the following graphs could represent $f(x) = x^4 + 2x^3 + 3x^2 - 5x - 6$?



(a) (b) (c) (d)

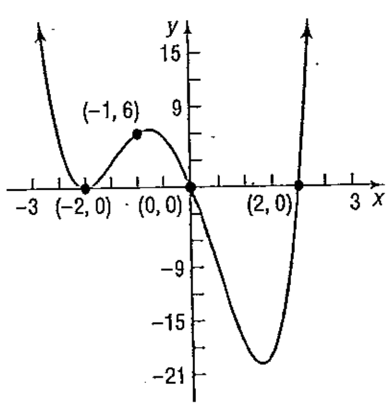
Think about:

- What is the y –intercept? (set $x=0$) y-int = -6
- What is the end behavior? like x^4 ↑↑
- What is the maximum number of turns? $n-1 = 4-1 = 3$

The graph __b__ could be the graph of $f(x) = x^4 + 2x^3 + 3x^2 - 5x - 6$

WRITING A POLYNOMIAL FXN FROM ITS GRAPH

Write a polynomial function whose graph is show below. Use the smallest degree possible. [Hint: identify zeros, multiplicities, y intercept, find the leading coefficient using a given point on the graph].



zeros	multip	C/T
−2	even	T
0	odd	C
2	odd	C

y-int = 0

$f(x) = ax(x+2)^2(x-2)$

$n=3$ so degree is at most 4

use $(-1, 6)$
$6 = a(-1)(-1+2)^2(-1-2)$
$6 = a(-1)(1)(-3)$
$6 = a(3)$
$a = 2$

$\therefore f(x) = 2x(x+2)^2(x-2)$

<u>ANALYZE THE GRAPH OF A POLYNOMIAL FXN</u>

Step 1: Determine the end behavior of the graph of the function.
Step 2: Find the x- and y- intercepts of the graph of the function.
Step 3: Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.
Step 4: Determine the maximum number of turning points on the graph of the function.
Step 5: Use the information in Steps 1 through 4 to draw a complete graph of the function.

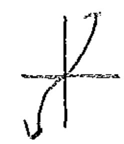
Analyze the graph of $f(x) = (2x+1)(x-3)^2$

Step 1: $f(x) = (2x+1)(x-3)^2$

$= (2x+1)(x^2-6x+9)$

$= 2x^3 - 12x^2 + 18x + x^2 - 6x + 9$

$= 2x^3 - 11x^2 + 12x + 9$



end behavior: x^3

$x \to \infty,\ f(x) \to \infty$

$x \to -\infty,\ f(x) \to -\infty$

Step 2: y int $= 9$

$0 = (2x+1)(x-3)^2$

$2x+1=0 \quad (x-3)^2=0$

$x = -1/2 \quad x = 3$

Step 3:

zero	multip	C/T
$-1/2$	1	C
3	2	T

Step 4: $n-1 = 3-1 = 2$

Step 5: