Pre Calc
NOTES

T 1/24/17

3.4 Properties of Rational Functions

A rational function takes the form $R(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions (and $q(x) \neq$ 0). Ratios of integers are called rational numbers. Hence, ratios of functions are called rational functions.

I. DOMAIN OF RATIONAL FUNCTIONS

To determine the domain of a rational function, recall that the denominator $\neq$ 0. Therefore, the solutions to $q(x) = 0$ are the restrictions on the domain.

Examples: Find the domain of the rational functions.

a. $R(x) = \frac{2x^2-4}{x+5}$ $\quad x+5 \neq 0$, $-5 \; -5$, $x \neq -5$ $\quad \{x \mid x \neq -5\}$ or $(-\infty, -5),(-5, \infty)$

b. $R(x) = \frac{1}{x^2-4}$ $\quad x^2-4 \neq 0$, $+4 \; +4$ $\quad x^2 = 4$, $x = \pm 2$ $\quad \{x \mid x \neq 2, x \neq -2\}$

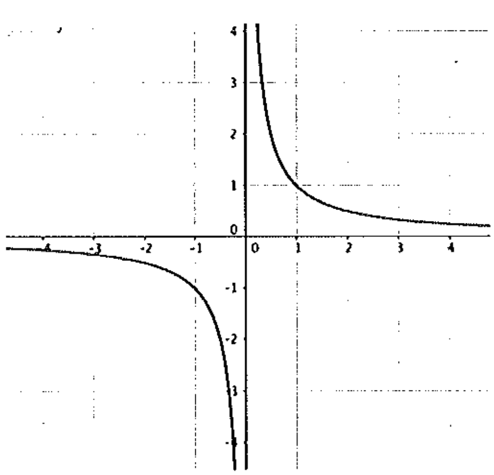
c. $R(x) = \frac{x^3}{x^2+1}$ $\quad x^2+1 \neq 0$, $x^2 \neq -1$ no solution ∴ no restriction on domain $\quad \{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

d. $R(x) = \frac{x^2-1}{x-1}$ $\quad x-1 \neq 0$, $x \neq 1$ $\quad \{x \mid x \neq 1\}$ or $(-\infty, 1)(1, \infty)$

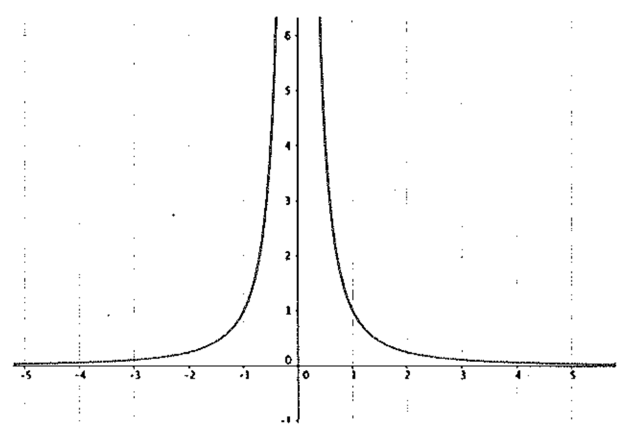
*Do not reduce a rational function when finding the domain.

II. USING TRANSFORMATIONS TO GRAPH A RATIONAL FUNCTION

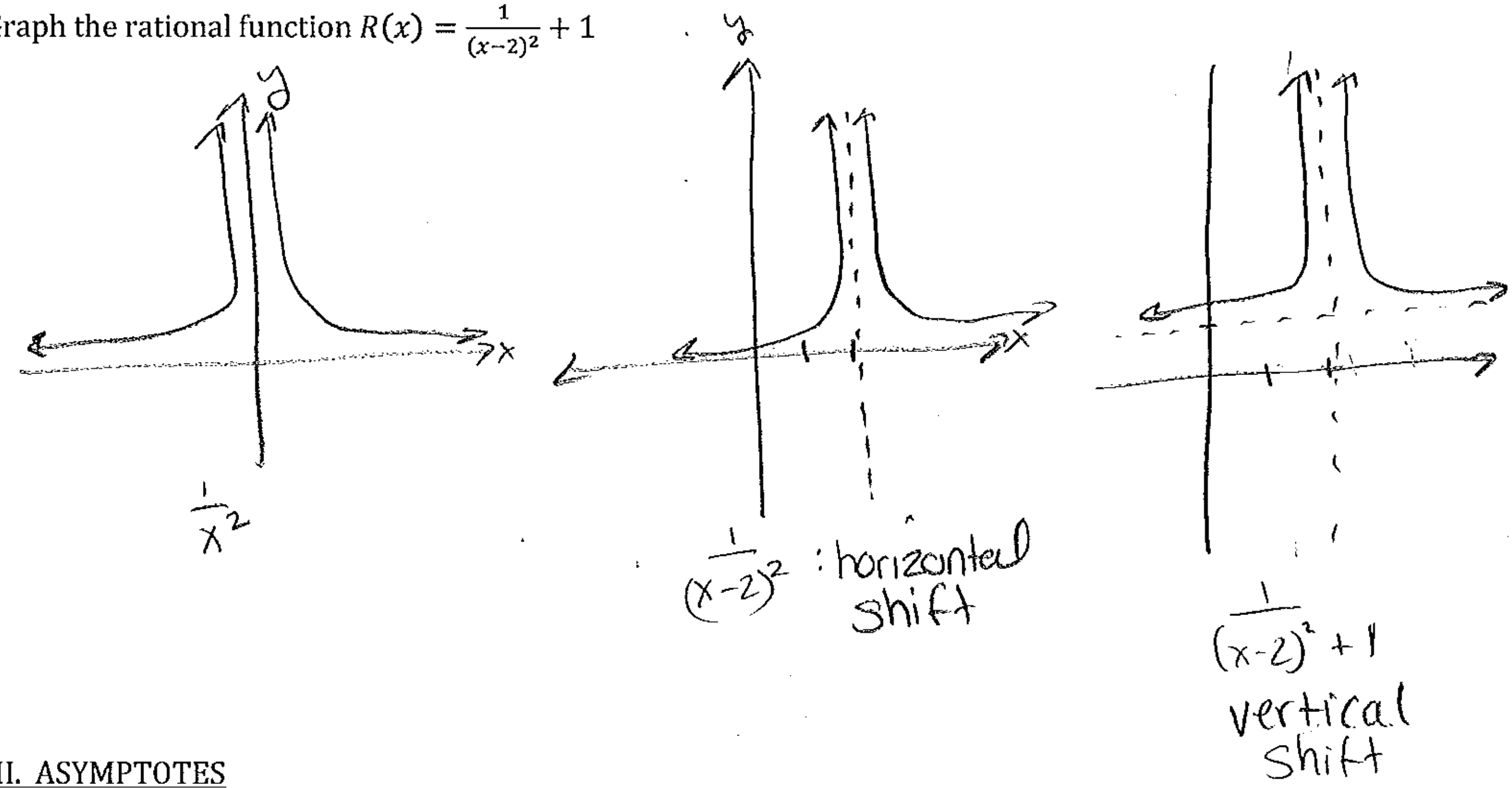
Graph of $f(x) = \frac{1}{x}$ "odd multiplicity"



Graph of $g(x) = \frac{1}{x^2}$ "even multiplicity"



Graph the rational function $R(x) = \frac{1}{(x-2)^2} + 1$



III. ASYMPTOTES

An asymptote is a straight line that a graph approaches but doesn't touch or cross. Rational functions have horizontal, vertical, and oblique asymptotes. Rational functions never cross vertical asymptotes, but sometimes cross horizontal asymptotes in the middle. Notice for the graph of $R(x) = \frac{1}{(x-2)^2} + 1$ above, as the graph approaches the vertical asymptote x = 2 from either the right (designated by 2^+) or the left (designated by 2^-), R(x) goes to infinity. In math notation, *as* $x \to 2^+, R(x) \to \infty$ *and as* $x \to 2^-$, $R(x) \to \infty$. Also, the graph of *R(x)* has a horizontal asymptote at y = 1. In math notation, *as* $x \to -\infty, R(x) \to 1$ *and as* $x \to \infty$, 1 $R(x) \to 1$.

FINDING AN ASYMPTOTE

Consider the general form of a rational function below.

$$R(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + a_1 x + a_0}$$

The degree of the numerator is n and the degree of the denominator is m.

HORIZONTAL ASYMPTOTES

DEGREE	TYPE OF ASYMP	EQN OF ASYMP	EXAMPLE
$n < m$	Horizontal	y = 0	$R(x) = \frac{x^2}{x^3+1}$ HA @ y=_0_
$n = m$	Horizontal	$y = \frac{a_n}{b_m}$	$R(x) = \frac{2x^4}{3x^4+1}$ HA @ y=_2/3_
$n = m + 1$	Oblique	y = ax+b *(using long division)*	X

VERTICAL ASYMPTOTES

A rational function, $R(x) = \frac{p(x)}{q(x)}$, in lowest terms, will have a vertical asymptote x= r, if r is a real zero of the _denominator, q_. That is, if *x-r* is a factor of the denominator *q* of a rational function $R(x) = \frac{p(x)}{q(x)}$, in lowest terms, R will have the vertical asymptote *x* = *r*.

Example. Find the vertical asymptote of

a) $G(x) = \frac{x^2-9}{x^2+4x-21} = \frac{(x+3)\cancel{(x-3)}}{(x+7)\cancel{(x-3)}}$

D: $\{x \mid x \neq 3\}$

$x+7=0$
$-7 \quad -7$
$x=-7$ is a vertical asym

b) $H(x) = \frac{x^2}{x^2+1}$

x^2+1 is not factorable
$\therefore$ no vertical asymp.