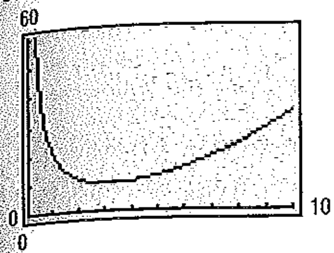
Figure 45



Substituting this expression for h, the cost C, in cents, as a function of the radius r is

$$C(r) = 0.10\pi r^2 + 0.04\pi r \cdot \frac{500}{\pi r^2} = 0.10\pi r^2 + \frac{20}{r} = \frac{0.10\pi r^3 + 20}{r}$$

(b) See Figure 45 for the graph of $C = C(r)$.

(c) Using the MINIMUM command, the cost is least for a radius of about 3.17 centimeters.

(d) The least cost is $C(3.17) \approx 9.47¢$.

Now Work PROBLEM 67

3.5 Assess Your Understanding

'Are You Prepared?' *The answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Find the intercepts of the graph of the equation $y = \frac{x^2 - 1}{x^2 - 4}$. (pp. 11–12)

2. Solve $\frac{x - 3}{x^2 + 1} = -2$. (pp. A66–A67)

Concepts and Vocabulary

3. If the numerator and the denominator of a rational function have no common factors, the rational function is ________.

4. The graph of a rational function never intersects a ________ asymptote.

5. ***True or False*** The graph of a rational function sometimes intersects an oblique asymptote.

6. ***True or False*** The graph of a rational function sometimes has a hole.

Skill Building

In Problems 7–50, follow Steps 1 through 7 on page 246 to analyze the graph of each function.

7. $R(x) = \frac{x + 1}{x(x + 4)}$

8. $R(x) = \frac{x}{(x - 1)(x + 2)}$

9. $R(x) = \frac{3x + 3}{2x + 4}$

10. $R(x) = \frac{2x + 4}{x - 1}$

11. $R(x) = \frac{3}{x^2 - 4}$

12. $R(x) = \frac{6}{x^2 - x - 6}$

13. $P(x) = \frac{x^4 + x^2 + 1}{x^2 - 1}$

14. $Q(x) = \frac{x^4 - 1}{x^2 - 4}$

15. $H(x) = \frac{x^3 - 1}{x^2 - 9}$

16. $G(x) = \frac{x^3 + 1}{x^2 + 2x}$

17. $R(x) = \frac{x^2}{x^2 + x - 6}$

18. $R(x) = \frac{x^2 + x - 12}{x^2 - 4}$

19. $G(x) = \frac{x}{x^2 - 4}$

20. $G(x) = \frac{3x}{x^2 - 1}$

21. $R(x) = \frac{3}{(x - 1)(x^2 - 4)}$

22. $R(x) = \frac{-4}{(x + 1)(x^2 - 9)}$

23. $H(x) = \frac{x^2 - 1}{x^4 - 16}$

24. $H(x) = \frac{x^2 + 4}{x^4 - 1}$

25. $F(x) = \frac{x^2 - 3x - 4}{x + 2}$

26. $F(x) = \frac{x^2 + 3x + 2}{x - 1}$

27. $R(x) = \frac{x^2 + x - 12}{x - 4}$

28. $R(x) = \frac{x^2 - x - 12}{x + 5}$

29. $F(x) = \frac{x^2 + x - 12}{x + 2}$

30. $G(x) = \frac{x^2 - x - 12}{x + 1}$

31. $R(x) = \frac{x(x - 1)^2}{(x + 3)^3}$

32. $R(x) = \frac{(x - 1)(x + 2)(x - 3)}{x(x - 4)^2}$

33. $R(x) = \frac{x^2 + x - 12}{x^2 - x - 6}$

34. $R(x) = \frac{x^2 + 3x - 10}{x^2 + 8x + 15}$

35. $R(x) = \frac{6x^2 - 7x - 3}{2x^2 - 7x + 6}$

36. $R(x) = \frac{8x^2 + 26x + 15}{2x^2 - x - 15}$

37. $R(x) = \frac{x^2 + 5x + 6}{x + 3}$

38. $R(x) = \frac{x^2 + x - 30}{x + 6}$

39. $H(x) = \frac{3x - 6}{4 - x^2}$

40. $H(x) = \dfrac{2 - 2x}{x^2 - 1}$

41. $F(x) = \dfrac{x^2 - 5x + 4}{x^2 - 2x + 1}$

42. $F(x) = \dfrac{x^2 - 2x - 15}{x^2 + 6x + 9}$

43. $G(x) = \dfrac{x}{(x + 2)^2}$

44. $G(x) = \dfrac{2 - x}{(x - 1)^2}$

45. $f(x) = x + \dfrac{1}{x}$

46. $f(x) = 2x + \dfrac{9}{x}$

47. $f(x) = x^2 + \dfrac{1}{x}$

48. $f(x) = 2x^2 + \dfrac{16}{x}$

49. $f(x) = x + \dfrac{1}{x^3}$

50. $f(x) = 2x + \dfrac{9}{x^3}$

In Problems 51–56, graph each function using a graphing utility; then use MINIMUM to obtain the minimum value, rounded to two decimal places.

51. $f(x) = x + \dfrac{1}{x}, \quad x > 0$

52. $f(x) = 2x + \dfrac{9}{x}, \quad x > 0$

53. $f(x) = x^2 + \dfrac{1}{x}, \quad x > 0$

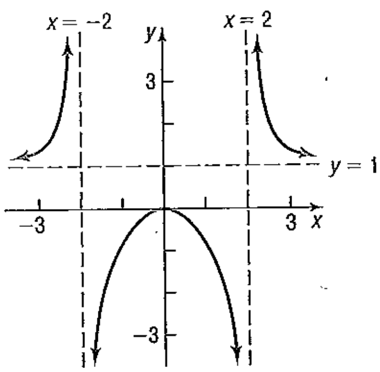
54. $f(x) = 2x^2 + \dfrac{9}{x}, \quad x > 0$

55. $f(x) = x + \dfrac{1}{x^3}, \quad x > 0$

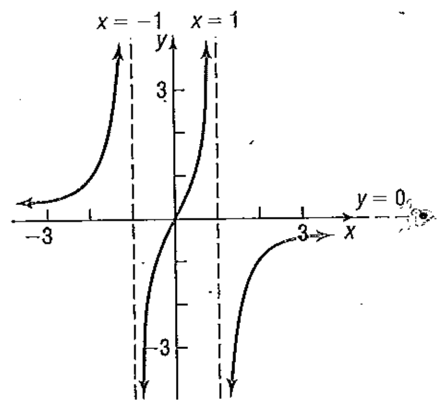
56. $f(x) = 2x + \dfrac{9}{x^3}, \quad x > 0$

In Problems 57–60, find a rational function that might have the given graph. (More than one answer might be possible.)

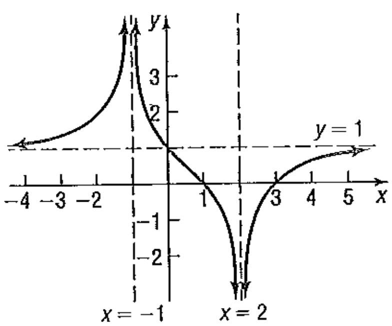
57.



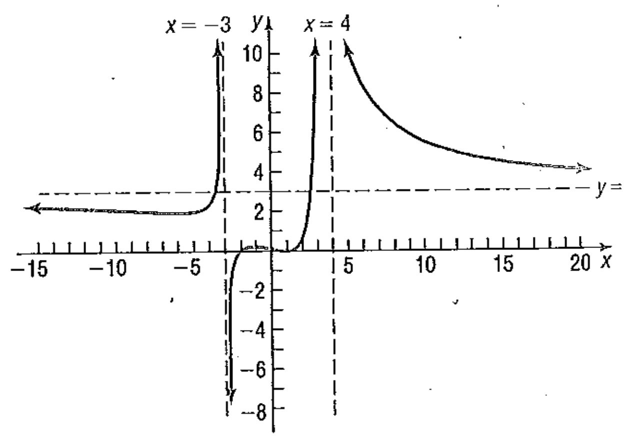
58.



59.



60.



Applications and Extensions

61. **Drug Concentration** The concentration C of a certain drug in a patient's bloodstream t hours after injection is given by

$$C(t) = \frac{t}{2t^2 + 1}$$

(a) Find the horizontal asymptote of $C(t)$. What happens to the concentration of the drug as t increases?

(b) Using your graphing utility, graph $C = C(t)$.

(c) Determine the time at which the concentration is highest.

62. **Drug Concentration** The concentration C of a certain drug in a patient's bloodstream t minutes after injection is given by

$$C(t) = \frac{50t}{t^2 + 25}$$

(a) Find the horizontal asymptote of $C(t)$. What happens to the concentration of the drug as t increases?

(b) Using your graphing utility, graph $C = C(t)$.

(c) Determine the time at which the concentration is highest.

63. **Minimum Cost** A rectangular area adjacent to a river is to be fenced in; no fence is needed on the river side. The enclosed area is to be 1000 square feet. Fencing for the side parallel to the river is \$5 per linear foot, and fencing for the other two sides is \$8 per linear foot; the four corner posts are \$25 apiece. Let x be the length of one of the sides perpendicular to the river.

(a) Write a function $C(x)$ that describes the cost of the project.

(b) What is the domain of C?

(c) Use a graphing utility to graph $C = C(x)$.

(d) Find the dimensions of the cheapest enclosure.

Source: http://dl.uncw.edu/digilib/mathematics/algebra/mat111hb/pandr/rational/rational.html

64. **Doppler Effect** The Doppler effect (named after Christian Doppler) is the change in the pitch (frequency) of the sound from a source (s) as heard by an observer (o) when one or both are in motion. If we assume both the source and the observer are moving in the same direction, the relationship is

$$f' = f_a\left(\frac{v - v_o}{v - v_s}\right)$$

where
f' = perceived pitch by the observer
f_a = actual pitch of the source
v = speed of sound in air (assume 772.4 mph)
v_o = speed of the observer
v_s = speed of the source

Suppose that you are traveling down the road at 45 mph and you hear an ambulance (with siren) coming toward you from the rear. The actual pitch of the siren is 600 hertz (Hz).

(a) Write a function $f'(v_s)$ that describes this scenario.

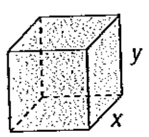
(b) If $f' = 620$ Hz, find the speed of the ambulance.

(c) Use a graphing utility to graph the function.

(d) Verify your answer from part (b).

Source: www.kettering.edu/~drussell/

65. **Minimizing Surface Area** United Parcel Service has contracted you to design a closed box with a square base that has a volume of 10,000 cubic inches. See the illustration.



(a) Express the surface area S of the box as a function of x.

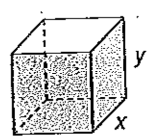
(b) Using a graphing utility, graph the function found in part (a).

(c) What is the minimum amount of cardboard that can be used to construct the box?

(d) What are the dimensions of the box that minimize the surface area?

(e) Why might UPS be interested in designing a box that minimizes the surface area?

66. **Minimizing Surface Area** United Parcel Service has contracted you to design an open box with a square base that has a volume of 5000 cubic inches. See the illustration.



(a) Express the surface area S of the box as a function of x.

(b) Using a graphing utility, graph the function found in part (a).

(c) What is the minimum amount of cardboard that can be used to construct the box?

(d) What are the dimensions of the box that minimize the surface area?

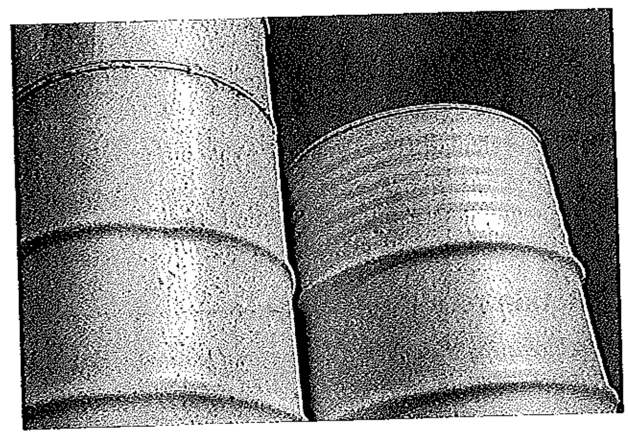
(e) Why might UPS be interested in designing a box that minimizes the surface area?

67. **Cost of a Can** A can in the shape of a right circular cylinder is required to have a volume of 500 cubic centimeters. The top and bottom are made of material that costs 6¢ per square centimeter, while the sides are made of material that costs 4¢ per square centimeter.

(a) Express the total cost C of the material as a function of the radius r of the cylinder. (Refer to Figure 44.)

(b) Graph $C = C(r)$. For what value of r is the cost C a minimum?

68. **Material Needed to Make a Drum** A steel drum in the shape of a right circular cylinder is required to have a volume of 100 cubic feet.



(a) Express the amount A of material required to make the drum as a function of the radius r of the cylinder.

(b) How much material is required if the drum's radius is 3 feet?

(c) How much material is required if the drum's radius is 4 feet?

(d) How much material is required if the drum's radius is 5 feet?

(e) Graph $A = A(r)$. For what value of r is A smallest?

69. **Removing a Discontinuity** In Example 5, we analyzed the rational function $R(x) = \dfrac{2x^2 - 5x + 2}{x^2 - 4}$. We found that the graph of the rational function has a hole at the point $\left(2, \frac{3}{4}\right)$. Therefore, the graph of R is discontinuous at $\left(2, \frac{3}{4}\right)$. We could remove this discontinuity by defining the rational function R using the following piecewise-defined function:

$$R(x) = \begin{cases} \dfrac{2x^2 - 5x + 2}{x^2 - 4} & \text{if } x \neq 2 \\ \dfrac{3}{4} & \text{if } x = 2 \end{cases}$$

(a) Redefine R from Problem 33 so that the discontinuity is removed.

(b) Redefine R from Problem 35 so that the discontinuity is removed.

70. Removing a Discontinuity See Problem 69.

(a) Redefine R from Problem 34 so that the discontinuity is removed.

(b) Redefine R from Problem 36 so that the discontinuity is removed.

Discussion and Writing

71. Graph each of the following functions:

$$y = \frac{x^2 - 1}{x - 1} \qquad y = \frac{x^3 - 1}{x - 1}$$

$$y = \frac{x^4 - 1}{x - 1} \qquad y = \frac{x^5 - 1}{x - 1}$$

Is $x = 1$ a vertical asymptote? Why not? What is happening for $x = 1$? What do you conjecture about $y = \frac{x^n - 1}{x - 1}$, $n \geq 1$ an integer, for $x = 1$?

72. Graph each of the following functions:

$$y = \frac{x^2}{x - 1} \quad y = \frac{x^4}{x - 1} \quad y = \frac{x^6}{x - 1} \quad y = \frac{x^8}{x - 1}$$

What similarities do you see? What differences?

73. Write a few paragraphs that provide a general strategy for graphing a rational function. Be sure to mention the following: proper, improper, intercepts, and asymptotes.

74. Create a rational function that has the following characteristics: crosses the x-axis at 2; touches the x-axis at -1; one vertical asymptote at $x = -5$ and another at $x = 6$; and one horizontal asymptote, $y = 3$. Compare your function to a fellow classmate's. How do they differ? What are their similarities?

75. Create a rational function that has the following characteristics: crosses the x-axis at 3; touches the x-axis at -2; one vertical asymptote, $x = 1$; and one horizontal asymptote, $y = 2$. Give your rational function to a fellow classmate and ask for a written critique of your rational function.

76. Create a rational function with the following characteristics: three real zeros, one of multiplicity 2; y-intercept, 1; vertical asymptotes, $x = -2$ and $x = 3$; oblique asymptote, $y = 2x + 1$. Is this rational function unique? Compare your function with those of other students. What will be the same as everyone else's? Add some more characteristics, such as symmetry or naming the real zeros. How does this modify the rational function?

77. Explain the circumstances under which the graph of a rational function will have a hole.

Retain Your Knowledge

Problems 78–81 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

78. Subtract: $(4x^3 - 7x + 1) - (5x^2 - 9x + 3)$

79. Solve: $\dfrac{3x}{3x + 1} = \dfrac{x - 2}{x + 5}$

80. Find the maximum value of $f(x) = -\dfrac{2}{3}x^2 + 6x - 5$.

81. Approximate $\dfrac{\sqrt{5} - 3}{\sqrt{7} + 2}$. Round your answer to three decimal places.

'Are You Prepared?' Answers

1. $\left(0, \frac{1}{4}\right), (1, 0), (-1, 0)$

2. $\left\{-1, \frac{1}{2}\right\}$

3.6 Polynomial and Rational Inequalities

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Solving Linear Inequalities (Appendix A, Section A.10, pp. A84–A86)
- Solving Quadratic Inequalities (Section 2.5, pp. 160–162)

Now Work the **'Are You Prepared?'** problems on page 263.

OBJECTIVES 1 Solve Polynomial Inequalities (p. 259)
2 Solve Rational Inequalities (p. 260)

(a) Redefine R from Problem 33 so that the discontinuity is removed.

(b) Redefine R from Problem 35 so that the discontinuity is removed.

70. Removing a Discontinuity See Problem 69.

(a) Redefine R from Problem 34 so that the discontinuity is removed.

(b) Redefine R from Problem 36 so that the discontinuity is removed.

Discussion and Writing

71. Graph each of the following functions:

$$y = \frac{x^2 - 1}{x - 1} \quad y = \frac{x^3 - 1}{x - 1}$$
$$y = \frac{x^4 - 1}{x - 1} \quad y = \frac{x^5 - 1}{x - 1}$$

Is $x = 1$ a vertical asymptote? Why not? What is happening for $x = 1$? What do you conjecture about $y = \frac{x^n - 1}{x - 1}$, $n \geq 1$ an integer, for $x = 1$?

72. Graph each of the following functions:

$$y = \frac{x^2}{x - 1} \quad y = \frac{x^4}{x - 1} \quad y = \frac{x^6}{x - 1} \quad y = \frac{x^8}{x - 1}$$

What similarities do you see? What differences?

73. Write a few paragraphs that provide a general strategy for graphing a rational function. Be sure to mention the following: proper, improper, intercepts, and asymptotes.

74. Create a rational function that has the following characteristics: crosses the x-axis at 2; touches the x-axis at -1; one vertical asymptote at $x = -5$ and another at $x = 6$; and one horizontal asymptote, $y = 3$. Compare your function to a fellow classmate's. How do they differ? What are their similarities?

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76. Create a rational function with the following characteristics: three real zeros, one of multiplicity 2; y-intercept, 1; vertical asymptotes, $x = -2$ and $x = 3$; oblique asymptote, $y = 2x + 1$. Is this rational function unique? Compare your function with those of other students. What will be the same as everyone else's? Add some more characteristics, such as symmetry or naming the real zeros. How does this modify the rational function?

77. Explain the circumstances under which the graph of a rational function will have a hole.

Retain Your Knowledge

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79. Solve: $\frac{3x}{3x + 1} = \frac{x - 2}{x + 5}$

80. Find the maximum value of $f(x) = -\frac{2}{3}x^2 + 6x - 5$.

81. Approximate $\frac{\sqrt{5} - 3}{\sqrt{7} + 2}$. Round your answer to three decimal places.

'Are You Prepared?' Answers

1. $\left(0, \frac{1}{4}\right), (1, 0), (-1, 0)$ **2.** $\left\{-1, \frac{1}{2}\right\}$

3.6 Polynomial and Rational Inequalities

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Solving Linear Inequalities (Appendix A, Section A.10, pp. A84–A86)
- Solving Quadratic Inequalities (Section 2.5, pp. 160–162)

Now Work the 'Are You Prepared?' problems on page 263.

OBJECTIVES 1 Solve Polynomial Inequalities (p. 259)
2 Solve Rational Inequalities (p. 260)