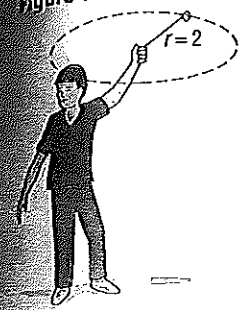
Figure 17



From equation (11), the linear speed v of the rock is

$$v = r\omega = 2 \text{ feet} \cdot 360\pi \frac{\text{radians}}{\text{minute}} = 720\pi \frac{\text{feet}}{\text{minute}} \approx 2262 \frac{\text{feet}}{\text{minute}}$$

The linear speed of the rock when it is released is 2262 ft/min $\approx$ 25.7 mi/hr.

Now Work PROBLEM 99

Historical Feature

Trigonometry was developed by Greek astronomers, who regarded the sky as the inside of a sphere, so it was natural that triangles on a sphere were investigated early (by Menelaus of Alexandria about AD 100) and that triangles in the plane were studied much later. The first book containing a systematic treatment of plane and spherical trigonometry was written by the Persian astronomer Nasir Eddin (about AD 1250).

Regiomontanus (1436–1476) is the person most responsible for moving trigonometry from astronomy into mathematics. His work was improved by Copernicus (1473–1543) and Copernicus's student Rhaeticus (1514–1576). Rhaeticus's book was the first to define the six trigonometric functions as ratios of sides of triangles, although he did not give the functions their present names. Credit for this is due to Thomas Finck (1583), but Finck's notation was by no means universally accepted at the time. The notation was finally stabilized by the textbooks of Leonhard Euler (1707–1783).

Trigonometry has since evolved from its use by surveyors, navigators, and engineers to present applications involving ocean tides, the rise and fall of food supplies in certain ecologies, brain wave patterns, and many other phenomena.

5.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. What is the formula for the circumference C of a circle of radius r? What is the formula for the area A of a circle of radius r? (p. A15)
2. If a particle has a speed of r feet per second and travels a distance d (in feet) in time t (in seconds), then d = _____. (pp. A75–A77)

Concepts and Vocabulary

3. An angle θ is in ________ ________ if its vertex is at the origin of a rectangular coordinate system and its initial side coincides with the positive x-axis.
4. A ________ ________ is a positive angle whose vertex is at the center of a circle.
5. If the radius of a circle is r and the length of the arc subtended by a central angle is also r, then the measure of the angle is 1 ________.
6. On a circle of radius r, a central angle of θ radians subtends an arc of length s = ________; the area of the sector formed by this angle θ is A = ________
7. 180° = ________ radians
8. An object travels around a circle of radius r with constant speed. If s is the distance traveled in time t around the circle and θ is the central angle (in radians) swept out in time t, then the linear speed of the object is v = ________ and the angular speed of the object is ω = ________.
9. *True or False* The angular speed ω of an object traveling around a circle of radius r is the angle θ (measured in radians) swept out, divided by the elapsed time t.
10. *True or False* For circular motion on a circle of radius r, linear speed equals angular speed divided by r.

Skill Building

In Problems 11–22, draw each angle in standard position.

11. 30°
12. 60°
13. 135°
14. −120°
15. 450°
16. 540°
17. $\frac{3\pi}{4}$
18. $\frac{4\pi}{3}$
19. $-\frac{\pi}{6}$
20. $-\frac{2\pi}{3}$
21. $\frac{16\pi}{3}$
22. $\frac{21\pi}{4}$

In Problems 23–28, convert each angle to a decimal in degrees. Round your answer to two decimal places.

23. $40°10'25''$ **24.** $61°42'21''$ **25.** $1°2'3''$ **26.** $73°40'40''$ **27.** $9°9'9''$ **28.** $98°22'45''$

In Problems 29–34, convert each angle to D°M'S" form. Round your answer to the nearest second.

29. $40.32°$ **30.** $61.24°$ **31.** $18.255°$ **32.** $29.411°$ **33.** $19.99°$ **34.** $44.01°$

In Problems 35–46, convert each angle in degrees to radians. Express your answer as a multiple of π.

35. $30°$ **36.** $120°$ **37.** $240°$ **38.** $330°$ **39.** $-60°$ **40.** $-30°$

41. $180°$ **42.** $270°$ **43.** $-135°$ **44.** $-225°$ **45.** $-90°$ **46.** $-180°$

In Problems 47–58, convert each angle in radians to degrees.

47. $\frac{\pi}{3}$ **48.** $\frac{5\pi}{6}$ **49.** $-\frac{5\pi}{4}$ **50.** $-\frac{2\pi}{3}$ **51.** $\frac{\pi}{2}$ **52.** 4π

53. $\frac{\pi}{12}$ **54.** $\frac{5\pi}{12}$ **55.** $-\frac{\pi}{2}$ **56.** $-\pi$ **57.** $-\frac{\pi}{6}$ **58.** $-\frac{3\pi}{4}$

In Problems 59–64, convert each angle in degrees to radians. Express your answer in decimal form, rounded to two decimal places.

59. $17°$ **60.** $73°$ **61.** $-40°$ **62.** $-51°$ **63.** $125°$ **64.** $350°$

In Problems 65–70, convert each angle in radians to degrees. Express your answer in decimal form, rounded to two decimal places.

65. 3.14 **66.** 0.75 **67.** 2 **68.** 3 **69.** 6.32 **70.** $\sqrt{2}$

In Problems 71–78, s denotes the length of the arc of a circle of radius r subtended by the central angle θ. Find the missing quantity. Round answers to three decimal places.

71. $r = 10$ meters, $\theta = \frac{1}{2}$ radian, $s = ?$

72. $r = 6$ feet, $\theta = 2$ radians, $s = ?$

73. $\theta = \frac{1}{3}$ radian, $s = 2$ feet, $r = ?$

74. $\theta = \frac{1}{4}$ radian, $s = 6$ centimeters, $r = ?$

75. $r = 5$ miles, $s = 3$ miles, $\theta = ?$

76. $r = 6$ meters, $s = 8$ meters, $\theta = ?$

77. $r = 2$ inches, $\theta = 30°$, $s = ?$

78. $r = 3$ meters, $\theta = 120°$, $s = ?$

In Problems 79–86, A denotes the area of the sector of a circle of radius r formed by the central angle θ. Find the missing quantity. Round answers to three decimal places.

79. $r = 10$ meters, $\theta = \frac{1}{2}$ radian, $A = ?$

80. $r = 6$ feet, $\theta = 2$ radians, $A = ?$

81. $\theta = \frac{1}{3}$ radian, $A = 2$ square feet, $r = ?$

82. $\theta = \frac{1}{4}$ radian, $A = 6$ square centimeters, $r = ?$

83. $r = 5$ miles, $A = 3$ square miles, $\theta = ?$

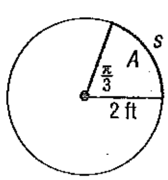
84. $r = 6$ meters, $A = 8$ square meters, $\theta = ?$

85. $r = 2$ inches, $\theta = 30°$, $A = ?$

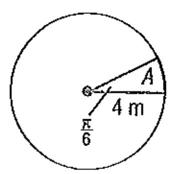
86. $r = 3$ meters, $\theta = 120°$, $A = ?$

In Problems 87–90, find the length s and area A. Round answers to three decimal places.

87.



88.



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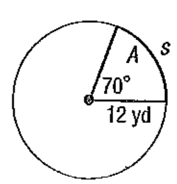

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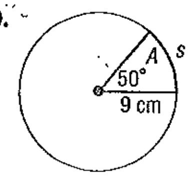
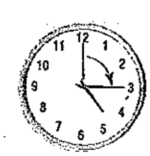


Applications and Extensions

91. Movement of a Minute Hand The minute hand of a clock is 6 inches long. How far does the tip of the minute hand move in 15 minutes? How far does it move in 25 minutes? Round answers to two decimal places.

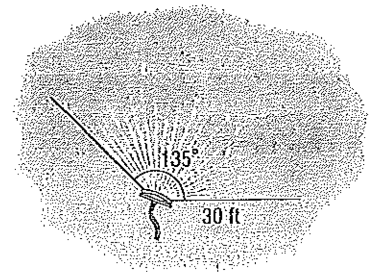


92. **Movement of a Pendulum** A pendulum swings through an angle of 20° each second. If the pendulum is 40 inches long, how far does its tip move each second? Round answers to two decimal places.

93. **Area of a Sector** Find the area of the sector of a circle of radius 4 meters formed by an angle of 45°. Round the answer to two decimal places.

94. **Area of a Sector** Find the area of the sector of a circle of radius 3 centimeters formed by an angle of 60°. Round the answer to two decimal places.

95. **Watering a Lawn** A water sprinkler sprays water over a distance of 30 feet while rotating through an angle of 135°. What area of lawn receives water?



96. **Designing a Water Sprinkler** An engineer is asked to design a water sprinkler that will cover a field of 100 square yards that is in the shape of a sector of a circle of radius 15 yards. Through what angle should the sprinkler rotate?

97. **Windshield Wiper** The arm and blade of a windshield wiper have a total length of 34 inches. If the blade is 25 inches long and the wiper sweeps out an angle of 120°, how much window area can the blade clean?

98. **Windshield Wiper** The arm and blade of a windshield wiper has a length of 30 inches. If the blade is 24 inches long and the wiper sweeps out an angle of 125°, how much window area can the blade clean?

99. **Motion on a Circle** An object is traveling around a circle with a radius of 5 centimeters. If in 20 seconds a central angle of $\frac{1}{3}$ radian is swept out, what is the angular speed of the object? What is its linear speed?

100. **Motion on a Circle** An object is traveling around a circle with a radius of 2 meters. If in 20 seconds the object travels 5 meters, what is its angular speed? What is its linear speed?

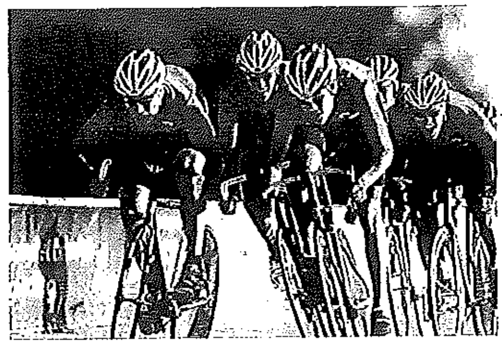
101. **Amusement Park Ride** A gondola on an amusement park ride, similar to the Spin Cycle at Silverwood Theme Park, spins at a speed of 13 revolutions per minute. If the gondola is 25 feet from the ride's center, what is the linear speed of the gondola in miles per hour?

102. **Amusement Park Ride** A centrifugal force ride, similar to the Gravitron, spins at a speed of 22 revolutions per minute. If the diameter of the ride is 13 meters, what is the linear speed of the passengers in kilometers per hour?

103. **Blu-Ray Drive** A Blu-ray drive has a maximum speed of 10,000 revolutions per minute. If a Blu-ray disc has a diameter of 12 cm, what is the linear speed, in km/hr, of a point 4 cm from the center if the disc is spinning at a rate of 8000 revolutions per minute?

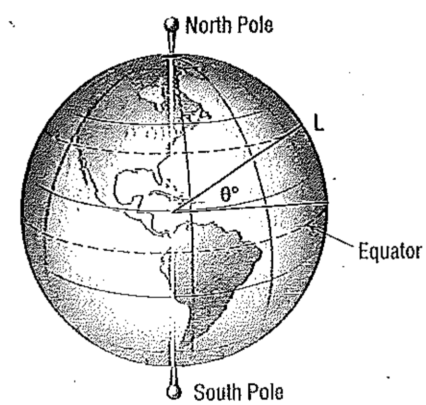
104. **DVD Drive** A DVD drive has a maximum speed of 7200 revolutions per minute. If a DVD has a diameter of 12 cm, what is the linear speed, in km/hr, of a point 5 cm from the disc's center if it is spinning at a rate of 5400 revolutions per minute?

105. **Bicycle Wheels** The diameter of each wheel of a bicycle is 26 inches. If you are traveling at a speed of 35 miles per hour on this bicycle, through how many revolutions per minute are the wheels turning?



106. **Car Wheels** The radius of each wheel of a car is 15 inches. If the wheels are turning at the rate of 3 revolutions per second, how fast is the car moving? Express your answer in inches per second and in miles per hour.

In Problems 107–110, the latitude of a location L is the angle formed by a ray drawn from the center of Earth to the equator and a ray drawn from the center of Earth to L. See the figure.



107. **Distance between Cities** Memphis, Tennessee, is due north of New Orleans, Louisiana. Find the distance between Memphis (35°9′ north latitude) and New Orleans (29°57′ north latitude). Assume that the radius of Earth is 3960 miles.

108. **Distance between Cities** Charleston, West Virginia, is due north of Jacksonville, Florida. Find the distance between Charleston (38°21′ north latitude) and Jacksonville (30°20′ north latitude). Assume that the radius of Earth is 3960 miles.

109. **Linear Speed on Earth** Earth rotates on an axis through its poles. The distance from the axis to a location on Earth 30° north latitude is about 3429.5 miles. Therefore, a location on Earth at 30° north latitude is spinning on a circle of radius 3429.5 miles. Compute the linear speed on the surface of Earth at 30° north latitude.

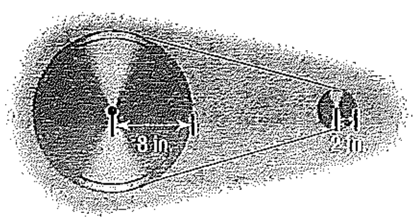
110. **Linear Speed on Earth** Earth rotates on an axis through its poles. The distance from the axis to a location on Earth 40° north latitude is about 3033.5 miles. Therefore, a location on Earth at 40° north latitude is spinning on a circle of radius 3033.5 miles. Compute the linear speed on the surface of Earth at 40° north latitude.

111. Speed of the Moon The mean distance of the moon from Earth is 2.39×10^5 miles. Assuming that the orbit of the moon around Earth is circular and that 1 revolution takes 27.3 days, find the linear speed of the moon. Express your answer in miles per hour.

112. Speed of Earth The mean distance of Earth from the Sun is 9.29×10^7 miles. Assuming that the orbit of Earth around the Sun is circular and that 1 revolution takes 365 days, find the linear speed of Earth. Express your answer in miles per hour.

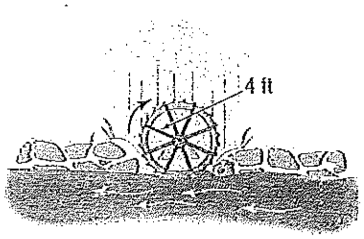
113. Pulleys Two pulleys, one with radius 2 inches and the other with radius 8 inches, are connected by a belt. (See the figure.) If the 2-inch pulley is caused to rotate at 3 revolutions per minute, determine the revolutions per minute of the 8-inch pulley.

[**Hint:** The linear speeds of the pulleys are the same; both equal the speed of the belt.]



114. Ferris Wheels A neighborhood carnival has a Ferris wheel whose radius is 30 feet. You measure the time it takes for one revolution to be 70 seconds. What is the linear speed (in feet per second) of this Ferris wheel? What is the angular speed in radians per second?

115. Computing the Speed of a River Current To approximate the speed of the current of a river, a circular paddle wheel with radius 4 feet is lowered into the water. If the current causes the wheel to rotate at a speed of 10 revolutions per minute, what is the speed of the current? Express your answer in miles per hour.

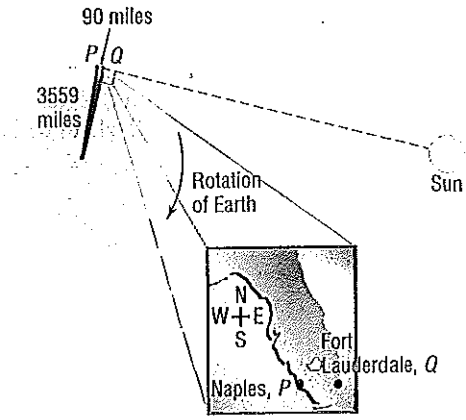


116. Spin Balancing Tires A spin balancer rotates the wheel of a car at 480 revolutions per minute. If the diameter of the wheel is 26 inches, what road speed is being tested? Express your answer in miles per hour. At how many revolutions per minute should the balancer be set to test a road speed of 80 miles per hour?

117. The Cable Cars of San Francisco At the Cable Car Museum you can see the four cable lines that are used to pull cable cars up and down the hills of San Francisco. Each cable travels at a speed of 9.55 miles per hour, caused by a rotating wheel whose diameter is 8.5 feet. How fast is the wheel rotating? Express your answer in revolutions per minute.

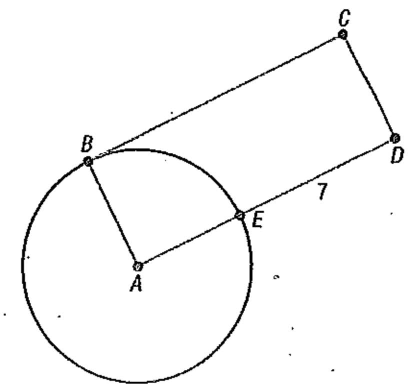
118. Difference in Time of Sunrise Naples, Florida is approximately 90 miles due west of Ft. Lauderdale. How much sooner would a person in Ft. Lauderdale first see the rising Sun than a person in Naples? See the hint.

[**Hint:** Consult the figure. When a person at Q sees the first rays of the Sun, a person at P is still in the dark. The person at P sees the first rays after Earth has rotated until P is at the location Q. Now use the fact that at the latitude of Ft. Lauderdale, in 24 hours an arc of length $2\pi(3559)$ miles is subtended.]



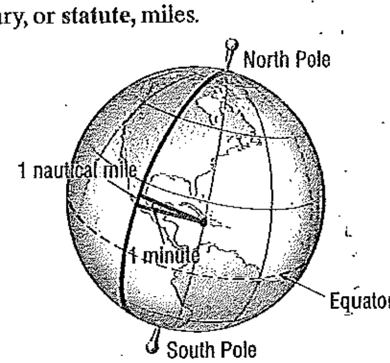
119. Let the Dog Roam A dog is attached to a 9-foot rope fastened to the outside corner of a fenced-in garden that measures 6 feet by 10 feet. Assuming that the dog cannot enter the garden, compute the exact area that the dog can wander. Write the exact area in square feet.*

120. Area of a Region The measure of arc $\widehat{BE}$ is 2π. Find the exact area of the portion of the rectangle $ABCD$ that falls outside of circle A.*



121. Keeping Up with the Sun How fast would you have to travel on the surface of Earth at the equator to keep up with the Sun (that is, so that the Sun would appear to remain in the same position in the sky)?

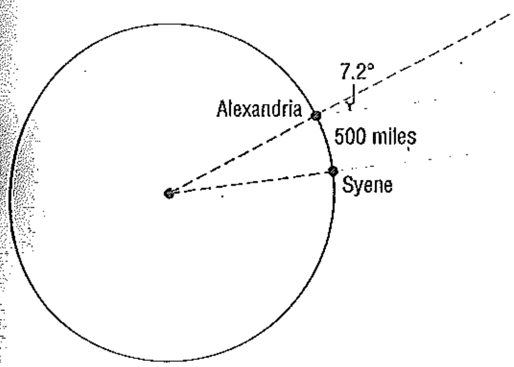
122. Nautical Miles A **nautical mile** equals the length of the arc subtended by a central angle of 1 minute on a great circle† on the surface of Earth. See the figure. If the radius of Earth is taken as 3960 miles, express 1 nautical mile in terms of ordinary, or **statute**, miles.



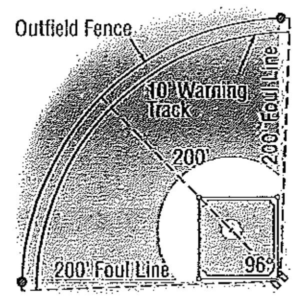
†Any circle drawn on the surface of Earth that divides Earth into two equal hemispheres.

*Courtesy of the Joliet Junior College Mathematics Department

123. **Approximating the Circumference of Earth** Eratosthenes of Cyrene (276–195 BC) was a Greek scholar who lived and worked in Cyrene and Alexandria. One day, while visiting in Syene, he noticed that the Sun's rays shone directly down a well. On this date 1 year later, in Alexandria, which is 500 miles due north of Syene, he measured the angle of the Sun to be about 7.2 degrees. See the figure. Use this information to approximate the radius and circumference of Earth.



124. **Designing a Little League Field** For a 60-foot Little League Baseball field, the distance from home base to the nearest fence (or other obstruction) in fair territory should be a minimum of 200 feet. The commissioner of parks and recreation is making plans for a new 60-foot field. Because of limited ground availability, he will use the minimum required distance to the outfield fence. To increase safety, however, he plans to include a 10-foot wide warning track on the inside of the fence. To further increase safety, the fence and warning track will extend both directions into foul territory. In total, the arc formed by the outfield fence (including the extensions into the foul territories) will be subtended by a central angle at home plate measuring 96°, as illustrated.
(a) Determine the length of the outfield fence.
(b) Determine the area of the warning track.



Source: www.littleleague.org

125. **Pulleys** Two pulleys, one with radius r_1 and the other with radius r_2, are connected by a belt. The pulley with radius r_1 rotates at ω_1 revolutions per minute, whereas the pulley with radius r_2 rotates at ω_2 revolutions per minute. Show that $\frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$.

Discussion and Writing

126. Do you prefer to measure angles using degrees or radians? Provide justification and a rationale for your choice.
127. What is 1 radian? What is 1 degree?
128. Which angle has the larger measure: 1 degree or 1 radian? Or are they equal?
129. Explain the difference between linear speed and angular speed.
130. For a circle of radius r, a central angle of θ degrees subtends an arc whose length s is $s = \frac{\pi}{180} r\theta$. Discuss whether this statement is true or false. Defend your position.
131. Discuss why ships and airplanes use nautical miles to measure distance. Explain the difference between a nautical mile and a statute mile.
132. Investigate the way that speed bicycles work. In particular, explain the differences and similarities between 5-speed and 9-speed derailleurs. Be sure to include a discussion of linear speed and angular speed.
133. In Example 6, we found that the distance between Dallas, Texas, and Sioux Falls, South Dakota, is approximately 744 miles. According to *mapquest.com*, the distance is approximately 850 miles. What might account for the difference?

Retain Your Knowledge

Problems 134–137 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

134. Find the zero of $f(x) = 3x + 7$.
135. Solve: $5x^2 + 2 = 5 - 14x$
136. Write the function that is finally graphed if all of the following transformations are applied to the graph of $y = |x|$.
(a) Shift left 3 units. (b) Reflect about the x-axis. (c) Shift down 4 units.
137. Find the horizontal and vertical asymptotes of $R(x) = \frac{3x^2 - 12}{x^2 - 5x - 14}$.

'Are You Prepared?' Answers

1. $C = 2\pi r;\ A = \pi r^2$ 2. $r \cdot t$