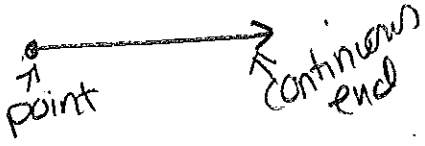
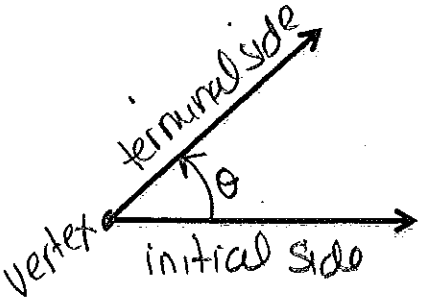
2/10/17

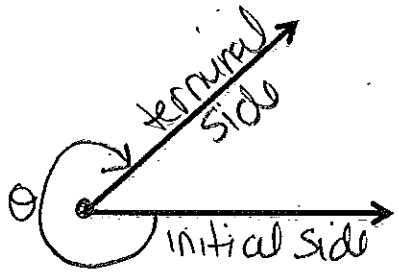
5.1 Angles & Their Measures NOTES

Ray: 

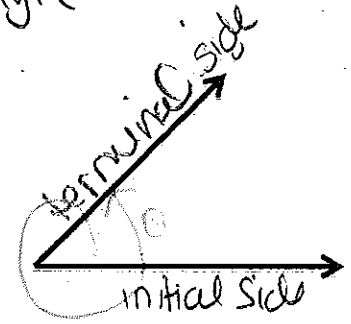
aka half line
two rays whose points meet create an angle



(+)angle
counterclockwise

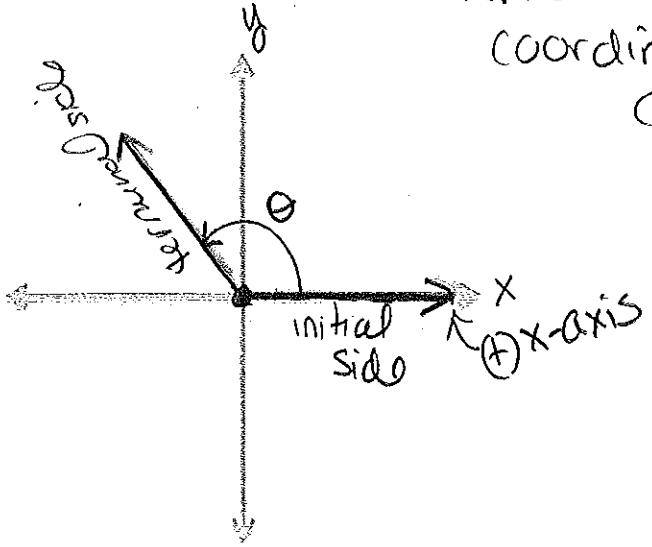


(-)angle
clockwise

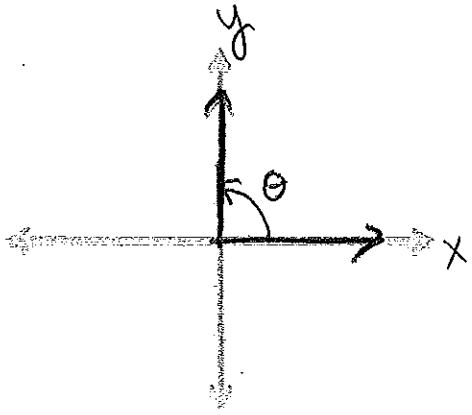


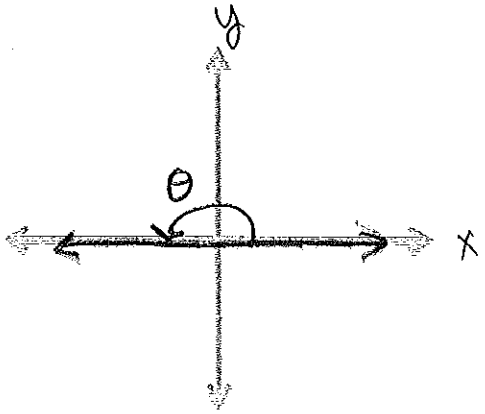
(+)angle
counterclockwise

ANGLES IN STANDARD POSITION: vertex is at the origin of rectangular coordinate system & initial side coincides with the positive x-axis.



QUADRANT ANGLES: when terminal side lays on the x- or y-axis.





DEGREES:

1 revolution = 360 °

$\frac{1}{360}$ revolution = 1 °

$\frac{1}{4}$ revolution = 90 °

$\frac{1}{2}$ revolution = 180 °

DRAW AN ANGLE

45° 90° 180° 45° 270°

-90° -90°

225° = 180° + 45° 225°

405° = 360° + 45° 405°

CONVERT BETWEEN DECIMALS AND DEGREES, MINUTES, SECONDS MEASURES FOR ANGLES

Degrees may be expressed as minutes and seconds. **One minute** is denoted by 1′ and is defined as $\frac{1}{60}$ degree. **One second** is denoted by 1″ and is defined as $\frac{1}{60}$ minute, or equivalently, $\frac{1}{3600}$ degree.

1 counterclockwise revolution = 360°
1° = 60′ 1′ = 60″

$1' = \left(\frac{1}{60}\right)^\circ$

$1'' = \left(\frac{1}{60}\right)^\circ \left(\frac{1}{60}\right)^\circ = \left(\frac{1}{3600}\right)^\circ$

Converting from degree, minute, second notation (D° M′ S″) to a decimal form and vice verse:

a. Convert 50°6′21″ to a decimal in degrees. Round to four decimal places.

$= 50^\circ + 6 \cdot 1' + 21 \cdot 1''$

$= 50^\circ + 6\left(\frac{1}{60}\right)^\circ + 21\left(\frac{1}{3600}\right)^\circ$

$\approx 50^\circ + .1^\circ + .0058^\circ$

$= 50.1058^\circ$

b. Convert 21.256° to the degree, minute, second (D° M′ S″) notation. Round the answer to the nearest second.

$21.256^\circ = 21^\circ + .256^\circ$

$= 21^\circ + .256 \cdot 1^\circ$ b/c $1^\circ = 60'$

$= 21^\circ + .256(60')$

$= 21^\circ + 15.36'$

$= 21^\circ + 15' + .36'$

$= 21^\circ + 15' + .36(60'')$

$= 21^\circ + 15' + 21.6''$

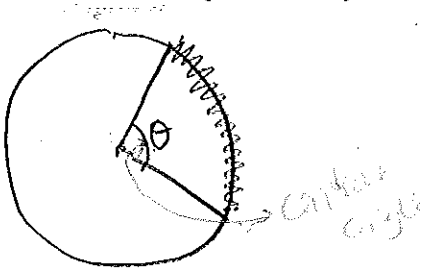
$= 21^\circ + 15' + 22''$

$= 21^\circ\ 15'\ 22''$

ARC LENGTH

For a circle of radius r, a central angle (vertex at center of circle) of θ radians subtends (intersects) an arc whose length s is defined as

$$s = r\theta$$



where θ is measured in radians.

Example. Find the arc length of a circle of radius 2 meters subtended by a central angle of 0.25 radian.

$r = 2$

$\theta = .25$

$S = r\theta$

$= (2)(.25)$

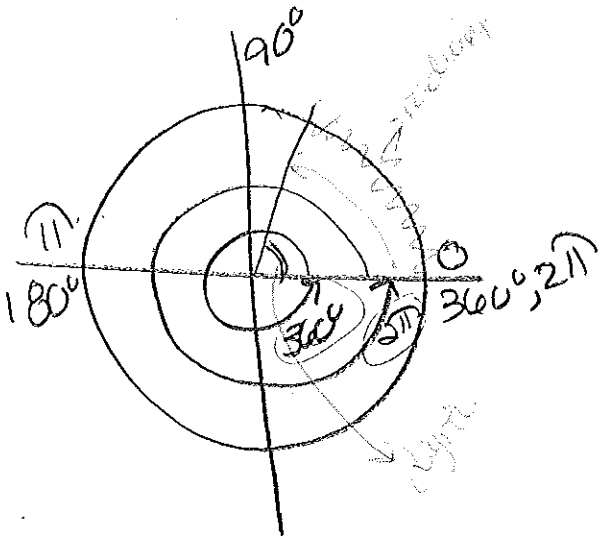
$= .5$ meters

CONVERT DEGREES TO RADIANS & RADIANS TO DEGREES

$360^\circ = 2\pi = 1 \text{ rev}$

$\frac{360^\circ}{2} = \frac{2\pi}{2}$

$180^\circ = \pi$



4^c

$\left(\frac{\pi}{2}\right)^c$

$\frac{180^\circ}{180^\circ} = \frac{\pi}{180}$

$1^\circ = \frac{\pi}{180^\circ}$

degree → radian

$\cdot \frac{\pi}{180^\circ}$

$\frac{180^\circ}{\pi} = \frac{\pi}{\pi}$

$\frac{180^\circ}{\pi} = 1 \text{ radian}$

$= 1^c$

radian → degree

$\cdot \frac{180^\circ}{\pi}$

$C = 2\pi r$

$C = 2\pi(1)$

$= 2\pi$

Example. Convert degrees to radians.

a) 60°

$\frac{60^\circ}{1} \cdot \frac{\pi}{180^\circ}$

$\frac{60\pi}{180} = \frac{6\pi}{18}$

$= \frac{\pi}{3}$

b) 150°

$\frac{150^\circ}{1} \cdot \frac{\pi}{180^\circ}$

$= \frac{150\pi}{180}$

$= \frac{15\pi}{18} = \frac{5\pi}{6}$

c) -45°

$-45^\circ \cdot \frac{\pi}{180^\circ}$

$-\frac{45\pi}{180_4}$

$= -\frac{\pi}{4}$

d) 90°

$\frac{90^\circ}{1} \cdot \frac{\pi}{180^\circ}$

$\frac{90\pi}{180_2}$

$= \frac{\pi}{2}$

e) 107°

$\frac{107^\circ}{1} \cdot \frac{\pi}{180^\circ}$

$\frac{107\pi}{180}$

$\approx 1.868 \text{ rad.}$

$\approx 1.868^c$

Example. Convert radian to degrees.

a) $\frac{\pi}{6}$

$\frac{\pi}{6} \cdot \frac{180^\circ}{\pi}$

$\frac{180^\circ}{6} = 30^\circ$

b) $\frac{3\pi}{2}$

$\frac{3\pi}{2_1} \cdot \frac{180^\circ}{\pi}$ (90°)

270°

c) $-\frac{3\pi}{4}$ (45°)

$-\frac{3\pi}{4} \cdot \frac{180^\circ}{\pi}$

-135°

d) 4 radians

$4 \cdot \frac{180^\circ}{\pi}$

$\approx 229.183^\circ$