5.2 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. In a right triangle, with legs a and b and hypotenuse c, the Pythagorean Theorem states that ________. (p. A14)
2. The value of the function $f(x) = 3x - 7$ at 5 is ___. (pp. 46–49)
3. ***True or False*** For a function $y = f(x)$, for each x in the domain, there is exactly one element y in the range. (pp. 43–46)
4. If two triangles are similar, then corresponding angles are _____ and the lengths of corresponding sides are ________. (p. A17)
5. What point is symmetric with respect to the y-axis to the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$? (pp. 12–14)
6. If (x, y) is a point on the unit circle in quadrant IV and if $x = \frac{\sqrt{3}}{2}$, what is y? (p. 34)

Concepts and Vocabulary

7. The _____ function takes as input a real number t that corresponds to a point $P = (x, y)$ on the unit circle and outputs the x-coordinate.
8. The point on the unit circle that corresponds to $\theta = \frac{\pi}{2}$ is $P =$ ____.
9. The point on the unit circle that corresponds to $\theta = \frac{\pi}{4}$ is

   $P =$ ________.
10. The point on the unit circle that corresponds to $\theta = \frac{\pi}{3}$ is

    $P =$ ________
11. For any angle θ in standard position, let $P = (x, y)$ be the point on the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$. Then, $\sin\theta =$ ____ and $\cos\theta =$ ____.
12. ***True or False*** Exact values can be found for the sine of any angle.

Skill Building

In Problems 13–20, $P = (x, y)$ is the point on the unit circle that corresponds to a real number t. Find the exact values of the six trigonometric functions of t.

13. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
14. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
15. $\left(-\frac{2}{5}, \frac{\sqrt{21}}{5}\right)$
16. $\left(-\frac{1}{5}, \frac{2\sqrt{6}}{5}\right)$
17. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
18. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
19. $\left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$
20. $\left(-\frac{\sqrt{5}}{3}, -\frac{2}{3}\right)$

In Problems 21–30, find the exact value. Do not use a calculator.

21. $\sin\frac{11\pi}{2}$
22. $\cos(7\pi)$
23. $\tan(6\pi)$
24. $\cot\frac{7\pi}{2}$
25. $\csc\frac{11\pi}{2}$
26. $\sec(8\pi)$
27. $\cos\left(-\frac{3\pi}{2}\right)$
28. $\sin(-3\pi)$
29. $\sec(-\pi)$
30. $\tan(-3\pi)$

In Problems 31–46, find the exact value of each expression. Do not use a calculator.

31. $\sin 45° + \cos 60°$
32. $\sin 30° - \cos 45°$
33. $\sin 90° + \tan 45°$
34. $\cos 180° - \sin 180°$
35. $\sin 45° \cos 45°$
36. $\tan 45° \cos 30°$
37. $\csc 45° \tan 60°$
38. $\sec 30° \cot 45°$
39. $4 \sin 90° - 3 \tan 180°$
40. $5 \cos 90° - 8 \sin 270°$
41. $2 \sin\frac{\pi}{3} - 3 \tan\frac{\pi}{6}$
42. $2 \sin\frac{\pi}{4} + 3 \tan\frac{\pi}{4}$
43. $2 \sec\frac{\pi}{4} + 4 \cot\frac{\pi}{3}$
44. $3 \csc\frac{\pi}{3} + \cot\frac{\pi}{4}$
45. $\csc\frac{\pi}{2} + \cot\frac{\pi}{2}$
46. $\sec\pi - \csc\frac{\pi}{2}$

In Problems 47–64, find the exact values of the six trigonometric functions of the given angle. If any are not defined, say "not defined." Do not use a calculator.

47. $\frac{2\pi}{3}$ **48.** $\frac{5\pi}{6}$ **49.** $210°$ **50.** $240°$ **51.** $\frac{3\pi}{4}$ **52.** $\frac{11\pi}{4}$

53. $\frac{8\pi}{3}$ **54.** $\frac{13\pi}{6}$ **55.** $405°$ **56.** $390°$ **57.** $-\frac{\pi}{6}$ **58.** $-\frac{\pi}{3}$

59. $-135°$ **60.** $-240°$ **61.** $\frac{5\pi}{2}$ **62.** 5π **63.** $-\frac{14\pi}{3}$ **64.** $-\frac{13\pi}{6}$

In Problems 65–76, use a calculator to find the approximate value of each expression rounded to two decimal places.

65. $\sin 28°$ **66.** $\cos 14°$ **67.** $\sec 21°$ **68.** $\cot 70°$

69. $\tan \frac{\pi}{10}$ **70.** $\sin \frac{\pi}{8}$ **71.** $\cot \frac{\pi}{12}$ **72.** $\csc \frac{5\pi}{13}$

73. $\sin 1$ **74.** $\tan 1$ **75.** $\sin 1°$ **76.** $\tan 1°$

In Problems 77–84, a point on the terminal side of an angle θ in standard position is given. Find the exact value of each of the six trigonometric functions of θ.

77. $(-3, 4)$ **78.** $(5, -12)$ **79.** $(2, -3)$ **80.** $(-1, -2)$

81. $(-2, -2)$ **82.** $(-1, 1)$ **83.** $\left(\frac{1}{3}, \frac{1}{4}\right)$ **84.** $(0.3, 0.4)$

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85. Find the exact value of:

$$\sin 45° + \sin 135° + \sin 225° + \sin 315°$$

86. Find the exact value of:

$$\tan 60° + \tan 150°$$

87. Find the exact value of:

$$\sin 40° + \sin 130° + \sin 220° + \sin 310°$$

88. Find the exact value of:

$$\tan 40° + \tan 140°$$

89. If $f(\theta) = \sin\theta = 0.1$, find $f(\theta + \pi)$.

90. If $f(\theta) = \cos\theta = 0.3$, find $f(\theta + \pi)$.

91. If $f(\theta) = \tan\theta = 3$, find $f(\theta + \pi)$.

92. If $f(\theta) = \cot\theta = -2$, find $f(\theta + \pi)$.

93. If $\sin\theta = \frac{1}{5}$, find $\csc\theta$.

94. If $\cos\theta = \frac{2}{3}$, find $\sec\theta$.

In Problems 95–106, $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$. Find the exact value of each function below if $\theta = 60°$. Do not use a calculator.

95. $f(\theta)$ **96.** $g(\theta)$ **97.** $f\left(\frac{\theta}{2}\right)$ **98.** $g\left(\frac{\theta}{2}\right)$

99. $[f(\theta)]^2$ **100.** $[g(\theta)]^2$ **101.** $f(2\theta)$ **102.** $g(2\theta)$

103. $2f(\theta)$ **104.** $2g(\theta)$ **105.** $f(-\theta)$ **106.** $g(-\theta)$

Mixed Practice

In Problems 107–116, $f(x) = \sin x$, $g(x) = \cos x$, $h(x) = 2x$, and $p(x) = \frac{x}{2}$. Find the value of each of the following:

107. $(f + g)(30°)$ **108.** $(f - g)(60°)$ **109.** $(f \cdot g)\left(\frac{3\pi}{4}\right)$ **110.** $(f \cdot g)\left(\frac{4\pi}{3}\right)$

111. $(f \circ h)\left(\frac{\pi}{6}\right)$ **112.** $(g \circ p)(60°)$ **113.** $(p \circ g)(315°)$ **114.** $(h \circ f)\left(\frac{5\pi}{6}\right)$

115. (a) Find $f\left(\frac{\pi}{4}\right)$. What point is on the graph of f?

(b) Assuming f is one-to-one*, use the result of part (a) to find a point on the graph of f^{-1}.

(c) What point is on the graph of $y = f\left(x + \frac{\pi}{4}\right) - 3$ if $x = \frac{\pi}{4}$?

116. (a) Find $g\left(\frac{\pi}{6}\right)$. What point is on the graph of g?

(b) Assuming g is one-to-one *, use the result of part (a) to find a point on the graph of g^{-1}.

(c) What point is on the graph of $y = 2g\left(x - \frac{\pi}{6}\right)$ if $x = \frac{\pi}{6}$?

In Section 7.1, we discuss the necessary domain restriction so that the function is one-to-one.

Applications and Extensions

117. Find two negative and three positive angles, expressed in radians, for which the point on the unit circle that corresponds to each angle is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

118. Find two negative and three positive angles, expressed in radians, for which the point on the unit circle that corresponds to each angle is $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

119. Use a calculator in radian mode to complete the following table.
What can you conclude about the value of $f(\theta) = \frac{\sin\theta}{\theta}$ as θ approaches 0?

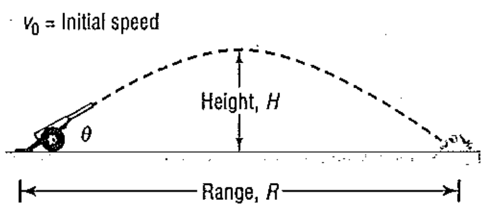
θ	0.5	0.4	0.2	0.1	0.01	0.001	0.0001	0.00001
$\sin\theta$								
$f(\theta) = \frac{\sin\theta}{\theta}$								

120. Use a calculator in radian mode to complete the following table.
What can you conclude about the value of $g(\theta) = \frac{\cos\theta - 1}{\theta}$ as θ approaches 0?

θ	0.5	0.4	0.2	0.1	0.01	0.001	0.0001	0.00001
$\cos\theta - 1$								
$g(\theta) = \frac{\cos\theta - 1}{\theta}$								

For Problems 121–124, use the following discussion.

Projectile Motion *The path of a projectile fired at an inclination θ to the horizontal with initial speed v_0 is a parabola (see the figure).*



The range R of the projectile—that is, the horizontal distance that the projectile travels—is found by using the function

$$R(\theta) = \frac{v_0^2 \sin(2\theta)}{g}$$

where $g \approx 32.2$ feet per second per second ≈ 9.8 meters per second per second is the acceleration due to gravity. The maximum height H of the projectile is given by the function

$$H(\theta) = \frac{v_0^2 (\sin\theta)^2}{2g}$$

In Problems 121–124, find the range R and maximum height H.

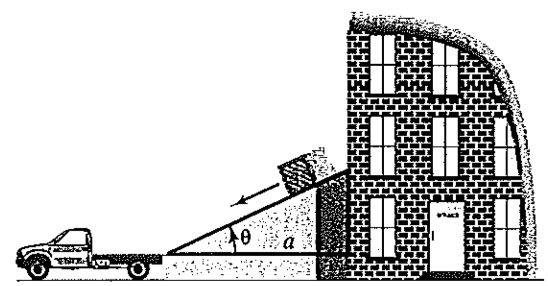
121. The projectile is fired at an angle of 45° to the horizontal with an initial speed of 100 feet per second.

122. The projectile is fired at an angle of 30° to the horizontal with an initial speed of 150 meters per second.

123. The projectile is fired at an angle of 25° to the horizontal with an initial speed of 500 meters per second.

124. The projectile is fired at an angle of 50° to the horizontal with an initial speed of 200 feet per second.

125. Inclined Plane See the figure.



If friction is ignored, the time t (in seconds) required for a block to slide down an inclined plane is given by the function

$$t(\theta) = \sqrt{\frac{2a}{g\sin\theta\cos\theta}}$$

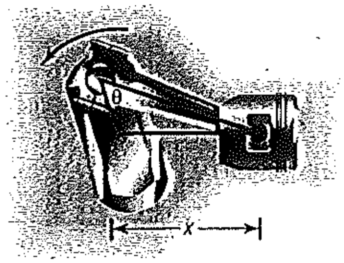
where a is the length (in feet) of the base and $g \approx 32$ feet per second per second is the acceleration due to gravity. How long does it take a block to slide down an inclined plane with base $a = 10$ feet when:

(a) $\theta = 30°$?
(b) $\theta = 45°$?
(c) $\theta = 60°$?

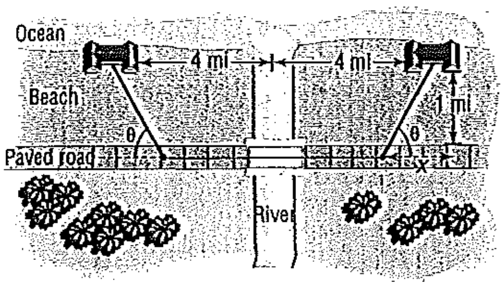
126. Piston Engines In a certain piston engine, the distance x (in centimeters) from the center of the drive shaft to the head of the piston is given by the function

$$x(\theta) = \cos\theta + \sqrt{16 + 0.5\cos(2\theta)}$$

where θ is the angle between the crank and the path of the piston head. See the figure. Find x when $\theta = 30°$ and when $\theta = 45°$.



127. Calculating the Time of a Trip Two oceanfront homes are located 8 miles apart on a straight stretch of beach, each a distance of 1 mile from a paved road that parallels the ocean. See the figure.



Sally can jog 8 miles per hour along the paved road, but only 3 miles per hour in the sand on the beach. Because of a river directly between the two houses, it is necessary to jog in the sand to the road, continue on the road, and then jog directly back in the sand to get from one house to the other. The time T to get from one house to the other as a function of the angle θ shown in the illustration is

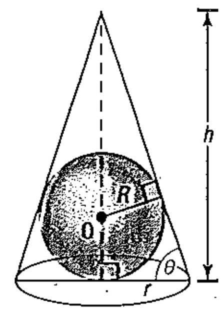
$$T(\theta) = 1 + \frac{2}{3\sin\theta} - \frac{1}{4\tan\theta}, \quad 0° < \theta < 90°$$

(a) Calculate the time T for $\theta = 30°$. How long is Sally on the paved road?
(b) Calculate the time T for $\theta = 45°$. How long is Sally on the paved road?
(c) Calculate the time T for $\theta = 60°$. How long is Sally on the paved road?
(d) Calculate the time T for $\theta = 90°$. Describe the path taken. Why can't the formula for T be used?

128. Designing Fine Decorative Pieces A designer of decorative art plans to market solid gold spheres encased in clear crystal cones. Each sphere is of fixed radius R and will be enclosed in a cone of height h and radius r. See the illustration. Many cones can be used to enclose the sphere, each having a different slant angle θ. The volume V of the cone can be expressed as a function of the slant angle θ of the cone as

$$V(\theta) = \frac{1}{3}\pi R^3 \frac{(1 + \sec\theta)^3}{(\tan\theta)^2}, \quad 0° < \theta < 90°$$

What volume V is required to enclose a sphere of radius 2 centimeters in a cone whose slant angle θ is 30°? 45°? 60°?



Use the following to answer Problems 129–132. The viewing angle, θ, of an object is the angle the object forms at the lens of the viewer's eye. This is also known as the perceived or angular size of the object. The viewing angle is related to the object's height, H, and distance from the viewer, D, through the formula $\tan\frac{\theta}{2} = \frac{H}{2D}$.

129. Tailgating While driving, Arletha observes the car in front of her with a viewing angle of 22°. If the car is 6 feet wide, how close is Arletha to the car in front of her? Round your answer to one decimal place.

130. Viewing Distance The Washington Monument in Washington, D.C. is 555 feet tall. If a tourist sees the monument with a viewing angle of 8°, how far away, to the nearest foot, is she from the monument?

131. Tree Height A forest ranger views a tree that is 200 feet away with a viewing angle of 20°. How tall is the tree to the nearest foot?

132. Radius of the Moon An astronomer observes the moon with a viewing angle of 0.52°. If the moon's average distance from Earth is 384,400 km, what is its radius to the nearest kilometer?

133. Let θ be the measure of an angle, in radians, in standard position with $\pi < \theta < \frac{3\pi}{2}$. Find the exact y-coordinate of the intersection of the terminal side of θ with the unit circle, given $\cos\theta + \sin^2\theta = \frac{41}{49}$. State the answer as a single fraction, completely simplified, with rationalized denominator.†

134. Let θ be the measure of an angle, in radians, in standard position with $\frac{\pi}{2} < \theta < \pi$. Find the exact x-coordinate of the intersection of the terminal side of θ with the unit circle, given $\cos^2\theta - \sin\theta = -\frac{1}{9}$. State the answer as a single fraction, completely simplified, with rationalized denominator.

135. Projectile Motion An object is propelled upward at an angle θ, $45° < \theta < 90°$, to the horizontal with an initial velocity of v_0 feet per second from the base of an inclined plane that makes an angle of 45° with the horizontal. See the illustration. If air resistance is ignored, the distance R that it travels up the inclined plane as a function of θ is given by

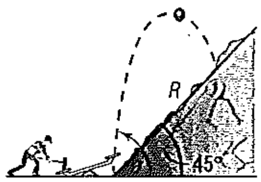
$$R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$$

(a) Find the distance R that the object travels along the inclined plane if the initial velocity is 32 feet per second and $\theta = 60°$.

†Courtesy of the Joliet Junior College Mathematics Department

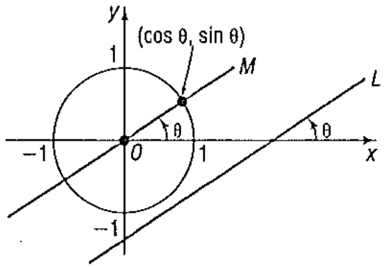
(b) Graph $R = R(\theta)$ if the initial velocity is 32 feet per second.

(c) What value of θ makes R largest?

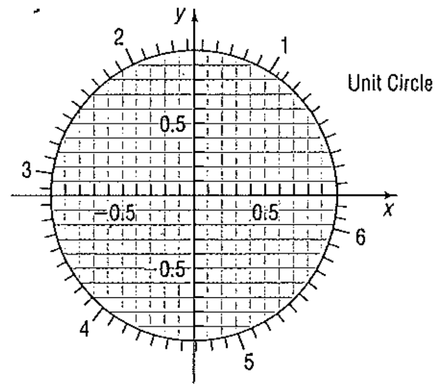


136. If θ, $0 < \theta < \pi$, is the angle between the positive x-axis and a nonhorizontal, nonvertical line L, show that the slope m of L equals $\tan \theta$. The angle θ is called the **inclination** of L.

[**Hint:** See the illustration, where we have drawn the line M parallel to L and passing through the origin. Use the fact that M intersects the unit circle at the point $(\cos \theta, \sin \theta)$.]



In Problems 137 and 138, use the figure to approximate the value of the six trigonometric functions at t to the nearest tenth. Then use a calculator to approximate each of the six trigonometric functions at t.



137. (a) $t = 1$

(b) $t = 5.1$

138. (a) $t = 2$

(b) $t = 4$

Discussion and Writing

139. Write a brief paragraph that explains how to quickly compute the trigonometric functions of 30°, 45°, and 60°.

140. Write a brief paragraph that explains how to quickly compute the trigonometric functions of 0°, 90°, 180°, and 270°.

141. How would you explain the meaning of the sine function to a fellow student who has just completed college algebra?

142. Draw a unit circle. Label the angles $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \ldots, \frac{7\pi}{4}, \frac{11\pi}{6}, 2\pi$ and the coordinates of the points on the unit circle that correspond to each of these angles. Explain how symmetry can be used to find the coordinates of points on the unit circle for angles whose terminal sides are in quadrants II, III, and IV.

Retain Your Knowledge

Problems 143–146 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

143. State the domain of $f(x) = \ln(5x + 2)$.

144. Given that the polynomial function $P(x) = x^4 - 5x^3 - 9x^2 + 155x - 250$ has zeros of $4 + 3i$ and 2, find the remaining zeros of the function.

145. Find the remainder when $P(x) = 8x^4 - 2x^3 + x - 8$ is divided by $x + 2$.

146. **Sidewalk Area** A sidewalk with a uniform width of 3 feet is to be placed around a circular garden with a diameter of 24 feet. Find the exact area of the sidewalk.

'Are You Prepared?' Answers

1. $c^2 = a^2 + b^2$ **2.** 8 **3.** True **4.** equal; proportional **5.** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **6.** $-\frac{1}{2}$