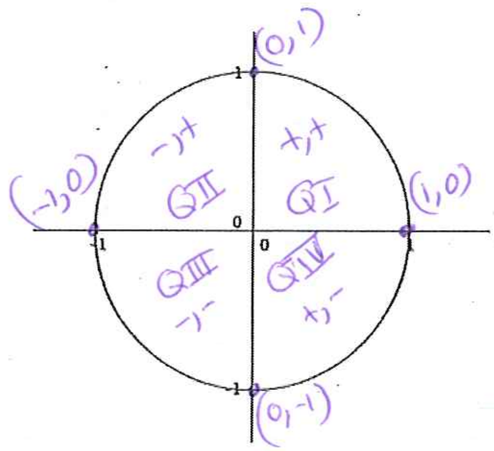
Calc

5.2 Trigonometric Functions: Unit Circle Approach

The unit circle has a radius = 1. In turn, the following points exist on the unit circle...



If the circumference C = $2\pi r$, then the circumference of the unit circle (whose radius is 1) is 2π. This implies that I revolution = 2π. So, a half revolution = π. A radian is the length of 1 radius. Since the unit circle has a circumference of 2π which is approximately 6.28, then the radius goes around the circumference of a circle ~6.28 times.

exactly 2π times

Let P be the point on the unit circle that corresponds to θ radians and let P = (x, y) be the point on the unit circle that corresponds to θ (radians). The six trigonometric functions may be expressed as....

$$\sin(\theta) = y \qquad \csc(\theta) = \frac{1}{y}$$

$$\cos(\theta) = x \qquad \sec(\theta) = \frac{1}{x}$$

$$\tan(\theta) = \frac{y}{x} \qquad \cot(\theta) = \frac{x}{y}$$

Notice that if x = 0, then tan(θ) and sec (θ) are undefined and if y = 0, then csc(θ) and cot(θ) are undefined.

FINDING THE EXACT VALUES OF THE TRIG FXNS USING A POINT ON THE UNIT CIRCLE

Let $P = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$ be the point on the unit circle that corresponds to θ (radians). Find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\csc(\theta)$, $\sec(\theta)$, and $\cot(\theta)$.

($x = -\frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$)

$\sin\theta = y = \frac{\sqrt{3}}{2}$

$\cos\theta = x = -\frac{1}{2}$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}/2}{-1/2} = \frac{\sqrt{3}}{2} \cdot -\frac{2}{1} = -\sqrt{3}$

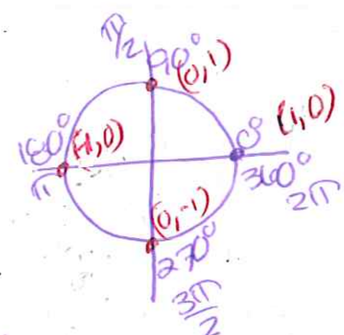
$\csc\theta = \frac{1}{y} = \frac{1}{\sqrt{3}/2} = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{1}{x} = \frac{1}{-1/2} = 1 \cdot -\frac{2}{1} = -2$

$\cot\theta = \frac{x}{y} = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$

$= -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

FINDING THE EXACT VALUES OF THE SIX TRIG FXNS OF **QUADRANT ANGLES**



a. $\theta = 0 = 0°$

$\sin(0°) = \sin(0) = 0$

$\cos(0°) = \cos(0) = 1$

$\tan(0°) = \tan(0) = \frac{0}{1} = 0$

$\csc(0°) = \csc(0) = \frac{1}{0}$ = undef.

$\sec(0°) = \sec(0) = \frac{1}{1} = 1$

$\cot(0°) = \cot(0) = \frac{1}{0}$ = undef

b. $\theta = \frac{\pi}{2} = 90°$

$\sin(90°) = \sin(\frac{\pi}{2}) = 1$

$\cos(90°) = \cos(\frac{\pi}{2}) = 0$

$\tan(90°) = \tan(\frac{\pi}{2}) = \frac{1}{0}$ = undef.

$\csc(90°) = \csc(\frac{\pi}{2}) = \frac{1}{1} = 1$

$\sec(90°) = \sec(\frac{\pi}{2}) = \frac{1}{0}$ = undef

$\cot(90°) = \cot(\frac{\pi}{2}) = \frac{0}{1} = 0$

c. $\theta = \pi = 180°$

$\sin(180°) = \sin(\pi) = 0$

$\cos(180°) = \cos(\pi) = -1$

$\tan(180°) = \tan(\pi) = \frac{0}{-1} = 0$

$\csc(180°) = \csc(\pi) = \frac{1}{0}$ undef

$\sec(180°) = \sec(\pi) = \frac{1}{-1} = -1$

$\cot(180°) = \cot(\pi) = \frac{-1}{0}$ undef

d. $\theta = \frac{3\pi}{2} = 270°$

$\sin(270°) = \sin(\frac{3\pi}{2}) = -1$

$\cos(270°) = \cos(\frac{3\pi}{2}) = 0$

$\tan(270°) = \tan(\frac{3\pi}{2}) = \frac{-1}{0}$ undef

$\csc(270°) = \csc(\frac{3\pi}{2}) = \frac{1}{-1} = -1$

$\sec(270°) = \sec(\frac{3\pi}{2}) = \frac{1}{0}$ undef

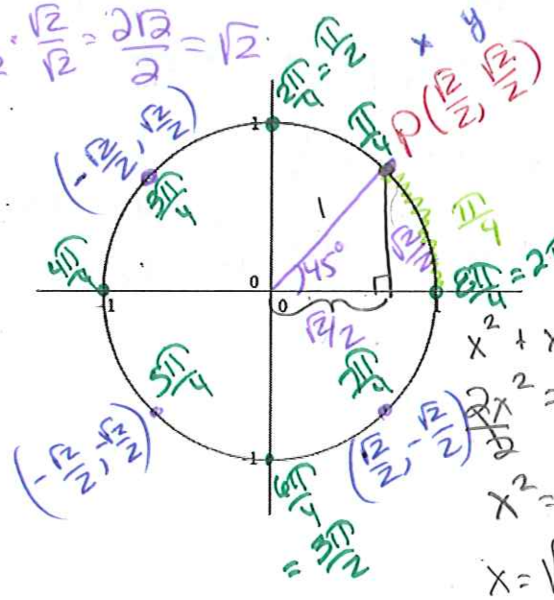
$\cot(270°) = \cot(\frac{3\pi}{2}) = \frac{0}{-1} = 0$

FIND THE EXACT VALUES OF THE TRIG FXNS OF $\frac{\pi}{4} = 45°$

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\quad \csc\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}/2} = 1 \cdot \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\quad \sec\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}/2} = \sqrt{2}$

$\tan\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$ $\quad \cot\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$



$x^2 + x^2 = 1^2$

$\frac{2x^2}{2} = \frac{1}{2}$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}}$

$x = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

FIND THE EXACT VALUE OF A TRIG EXPRESSION

a. $\sin(45°)\cos(180°)$ $= \left(\frac{\sqrt{2}}{2}\right)(-1) = -\frac{\sqrt{2}}{2}$

$\sin(45°) = \frac{\sqrt{2}}{2}$

$\cos(180°) = -1$

b. $\tan\left(\frac{\pi}{4}\right) - \sin\left(\frac{3\pi}{2}\right)$ $= 1 - (-1) = 2$

$\tan\left(\frac{\pi}{4}\right) = 1$

$\sin\left(\frac{3\pi}{2}\right) = -1$

c. $\left(\sec\frac{\pi}{4}\right)^2 + \csc\left(\frac{\pi}{2}\right)$ $= (\sqrt{2})^2 + 1 = 2 + 1 = 3$

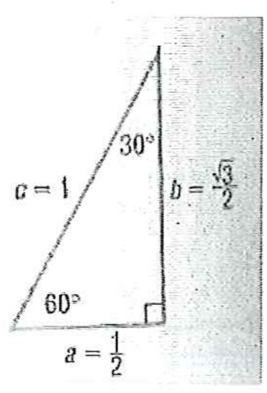
$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

$\csc\left(\frac{\pi}{2}\right) = 1$

FIND THE EXACT VALUES OF THE TRIG FXNS OF $\frac{\pi}{6} = 30°$ AND $\frac{\pi}{3} = 60°$

$\frac{\pi}{6} \cdot \frac{180°}{\pi} = \frac{180°}{6} = 30°$

Recall the special triangle 30-60-90

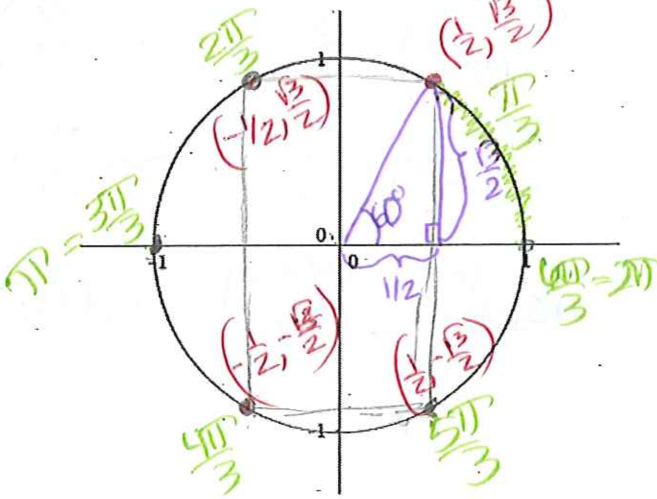


Find the exact values of the six trig fxns of $\frac{\pi}{3} = 60°$

$\sin(60°) = \sin\left(\frac{\pi}{3}\right) = \sqrt{3}/2$

$\cos(60°) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\tan(60°) = \tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3}$



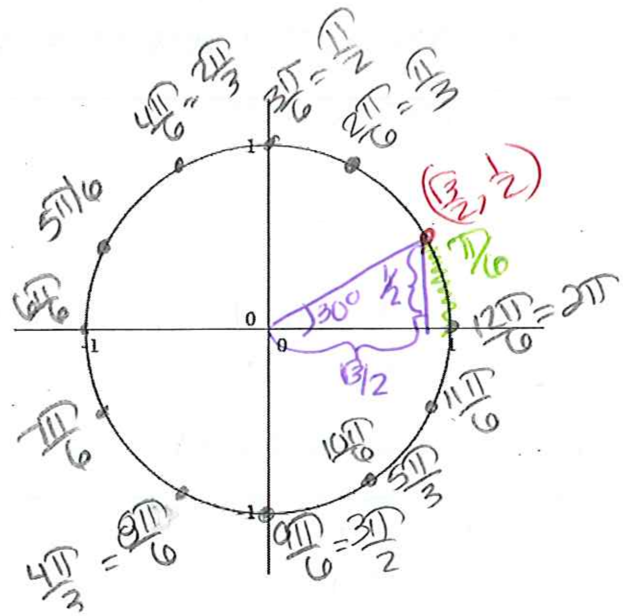
Find the exact values of the six trig fxns of $\frac{\pi}{6} = 30°$

$\sin(30°) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\cos(30°) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

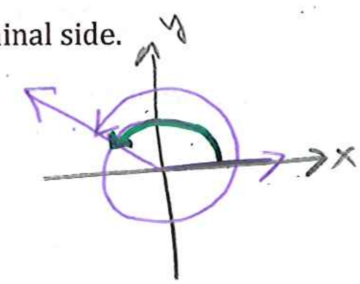
$\tan(30°) = \tan\left(\frac{\pi}{6}\right) = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$



COTERMINAL ANGLES

Coterminal angles are angles in standard position that have a common terminal side.



Example. 30°, -330°, and 390° are coterminal angles.

$30° + 360° = 390°$

$30° - 360° = -330°$

Example. $\frac{\pi}{2}$ and $\frac{5\pi}{2}$ are coterminal angles.

$\frac{\pi}{2} + 2\pi = \frac{\pi}{2} + \frac{4\pi}{2} = \frac{5\pi}{2}$

To find coterminal angles: degrees ± 360° and radians ± 2π