5.3 Conditional Probability

Learning Objectives

1. Calculate and interpret conditional probabilities.
2. Use the general multiplication rule to calculate probabilities.
3. Use tree diagrams to model a chance process and calculate probabilities involving two or more events.
4. Determine if two events are independent.
5. When appropriate, use the multiplication rule for independent events to compute probabilities.

Vocabulary: conditional probability, general multiplication rule, tree diagram, independent events, multiplication rule for independent events

Alternate Example: *Free Tacos!*
In 2012, fans at Arizona Diamondbacks home games would win 3 free tacos from Taco Bell if the Diamondbacks scored 6 or more runs. In the 2012 season, the Diamondbacks won 41 of their 81 home games and gave away free tacos in 30 of their 81 home games. In 26 of the games, the Diamondbacks won and gave away free tacos. Let W = win and T = free tacos. Choose a Diamondbacks home game at random.

(a) Summarize these data in a two-way table.

(b) Find the probability that the Diamondbacks win or there are free tacos.

(c) Find the probability that the Diamondbacks win, given that there are free tacos.

(d) Find the probability that there are free tacos, given that the Diamondbacks win.

The probability that one event happens given another event is already known to have happened is called ____. Suppose we know that event A has happened; then the probability that event B happens given that event A has happened is denoted by ________________.

Read 318–320

Alternate Example: *Late for School*

Shannon hits the snooze bar on her alarm clock on 60% of school days. If she doesn't hit the snooze bar, there is a 0.90 probability that she makes it to class on time. However, if she hits the snooze bar, there is only a 0.70 probability that she makes it to class on time.

(a) Use a tree diagram to represent this situation.

(b) On a randomly chosen day, what is the probability that Shannon is late for class?

(c) Suppose that Shannon is late for school. What is the probability that she hit the snooze bar that morning?

Read 321–326

<u>GENERAL MULTIPLICATION RULE</u>

When is it better to use a tree diagram than a two-way table?

Alternate Example: *False Positives and Drug Testing*
Many employers require prospective employees to take a drug test. A positive result on this test indicates that the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose that 4% of prospective employees use drugs, the false positive rate is 5%, and the false negative rate is 10%.

(a) Construct an appropriate diagram for this scenario. (table or tree?)

(b) What percent of prospective employees will test positive?

(c) What percent of prospective employees who test positive actually use illegal drugs?

Pierced Ears Example *continued*
Let's revisit the male vs. female with or without pierced ears scenario. Shown is the two-way table.

Pierced ears?	Male	Female	**Total**
Yes	19	84	**103**
No	71	4	**75**
Total	**90**	**88**	**178**

(a) What is the probability that a randomly selected person has pierced ears given that they are male? How would you denote that with symbols?

(b) Create a conditional distribution of the pierced ears for each gender and graph with a bar chart (from Chapter 1).

HS Grad & Homeowner *Example continued*. Below is the two-way table created in an earlier example that displays the results from a random sample of 500 people's high school graduation & homeowner status.

	HS Grad	Not HS Grad	**TOTAL**
Homeowner	221	119	**340**
NOT Homeowner	89	71	**160**
Total	**310**	**190**	**500**

Let G = HS Graduate
Let H = Homeowner

Find the following probabilities:

$P(G) =$

$P(H) =$

$P(G^c) =$

$P(H^c) =$

$P(G \cap H) =$

$P(G^c \cap H) =$

$P(G \mid H) =$

$P(H \mid G^c)$

Example: The Kaiser Family Foundation recently released a study about the influence of media in the lives of young people aged 8-18 (https://www.kff.org/other/event/generation-m2-media-in-the-lives-of/). In this study, 17% of the youth were classified as light media users, 62% were classified as moderate media users, and 21% were classified as heavy media users. Of the light users who responded, 74% described their grades as good (As and Bs), while only 68% of the moderate users and 52% of the heavy users described their grades as good. Suppose that we selected one young person a random.

(a) Draw a tree diagram to represent this situation.

(b) Find the probability that this person describes his or her grades as good.

(c) Given that this person describes his or her grades as good, what is the probability that he or she is a heavy user of media?

HW p. 317 (57–60), p. 333 (63–73 odd, 77, 79)

5.3 Conditional Probability and Independence

Read 326–328

Is there a relationship between gender and handedness? To find out, we used a CensusAtSchool's Random Data Selector to choose an SRS of 100 Australian high school students who completed a survey. The two way table displays data on gender and dominant hand of each student.

	Male	Female	**Total**
Right	39	51	**90**
Left	7	3	**10**
Total	**46**	**54**	**100**

Does knowing that a random student is male affect the probability of the student being left-handed? Justify your answer.

Two events *A* and *B* are _______________________________ if the occurrence of one event does not change the probability that the other event will happen.

For each chance process below, determine whether the events are independent. Justify your answer.

(a) Shuffle a standard deck of cards, and turn over the top card. Put it back in the deck, shuffle again, and turn over the top card. Define events A: first card is a heart, and B: second card is a heart.

(b) Shuffle a standard deck of cards, and turn over the top two cards, one at a time. Define events A: First card is a heart, and B: second card is a heart.

(c) 28 Students in an AP Statistics class completed a brief survey. One of the questions asked whether each student was right- or left-handed. The two way table summarizes the class data. Choose a student from the class at random. The events of interest are "female" and "right-handed."

	Female	Male
Left	3	1
Right	18	6

Example. Finger Length. Is there a relationship between gender and relative finger length? To find out, we used the random sampler at the United States CensusAtSchool website to randomly select 452 U.S. high school students who completed a survey. The two-way table shows the gender of each student and which finger was longer on their left hand (index finger or ring finger).

	Female	Male	**Total**
Index Finger	78	45	**123**
Ringer Finger	82	152	**234**
Same Length	52	43	**95**
Total	**212**	**240**	**452**

Are the events "female" and "has a longer ring finger" independent? Justify your answer

Read 328-331

What is the multiplication rule for independent events? Is it on the formula sheet? How is it related to the general multiplication rule?

What's the difference between "mutually exclusive" and "independent"?

Example: Choose a U.S. adult at random. Define event A: the person is male, and event B: the person is pregnant. Are the events mutually exclusive? Independent?

Example. According to Forrest Gump, "Life is like a box of chocolates. You never know what you're gonna get." Suppose a candy maker offers a special "Gump box" with 20 chocolates that look the same. In fact, 14 of the candies have soft centers and 6 have had centers. Choose 2 of the candies from a Gump box at random.

(a) Draw a tree diagram that shows the sample space of this chance process.

(b) Find the probability that one of the chocolates had a soft center AND the other one doesn't.

(c) Suppose you take 2 Gump boxes and choose 2 random chocolates from each. What is the probability that you get one soft center and one hard center from each?

(d) Suppose you take a sample of 100 boxes of Gump boxes and repeat this process again. What is the probability that you get one soft center and one hard center for all 100 boxes?

Alternate Example: *First Trimester Screen*
The First Trimester Screen is a non-invasive test given during the first trimester of pregnancy to determine if there are specific chromosomal abnormalities in the fetus. According to a study published in the New England Journal of Medicine in November 2005, approximately 5% of normal pregnancies will receive a false positive result. Among 100 women with normal pregnancies, what is the probability that there will be at least one false positive?

Alternate Example. *Lost Internet Sites*. Internet sites often vanish or move, so that references to them can't be followed. In fact, 13% of Internet sites referenced in major scientific journals are lost within two years after publication. If we randomly select seven Internet references, from scientific journals, what is the probability that at least one of them doesn't work two years later?

Alternate Example: *Weather Conditions.* On a recent day, weather.com forecast a 50% chance of rain in Portland and a 50% chance of rain in Gresham. What is the probability it will rain in both locations?

HW p 335 (81–93 odd, 97–99)