Pre Calc

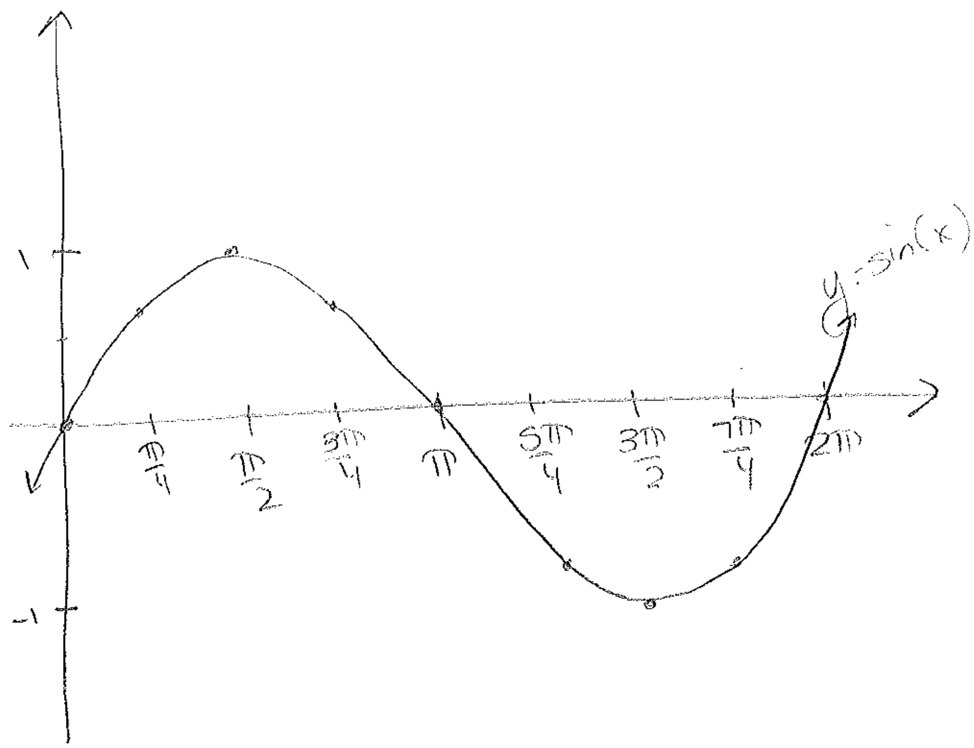
5.4 GRAPHS OF THE SINE AND COSINE FUNCTIONS

GRAPHING THE SINE FUNCTION

Since sine has a period of 2π, it is only necessary to graph y = sin(x) on the interval $[0, 2\pi]$ since the remainder of the graph is a repetition of this portion of the graph. To begin the process of graphing, let's fill out the table below. Then, graph the values with radians on the x-axis & sin(x) on the y-axis.

radian	sin(x)	approx
0	0	0
$\frac{\pi}{6}$	$\frac{1}{2}$	.5
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	.707
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	.866
$\frac{\pi}{2}$	1	1
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	.866
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	.707
$\frac{5\pi}{6}$	$\frac{1}{2}$	.5
π	0	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$	-.5
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	-.707
$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	-.866
$\frac{3\pi}{2}$	-1	-1
$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	-.866
$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	-.707
$\frac{11\pi}{6}$	$-\frac{1}{2}$	-.5
2π	0	0

Q1 Q2 Q3 Q4



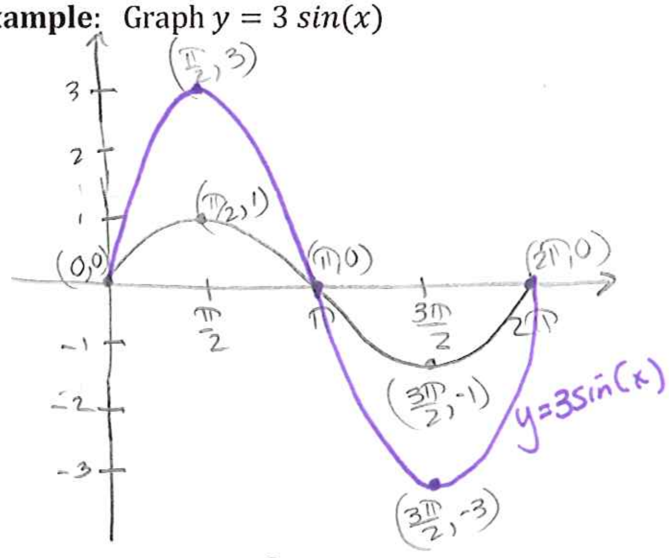
PROPERTIES OF THE SINE FUNCTION

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The sine function is an odd function.
4. The sine function is periodic with a period of 2π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi \ldots$; the y-intercept is 0.
6. The max value is 1 and occurs at $\ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$.
7. The min value is ______ and occurs at $\ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$.

GRAPH FUNCTIONS OF THE FORM $y = A \sin(\omega x)$

Use transformations to graph $y = A \sin(\omega x)$. A is the amplitude (or vertical dilation), where amplitude of the graph is the max height the graph reaches from the x-axis. A horizontal dilation is give by ω, which changes the period of the function. The period of the function is given by $\frac{2\pi}{\omega}$

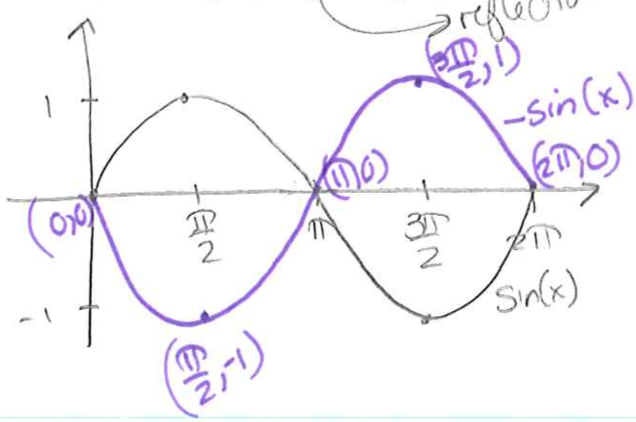
Example: Graph $y = 3\ sin(x)$



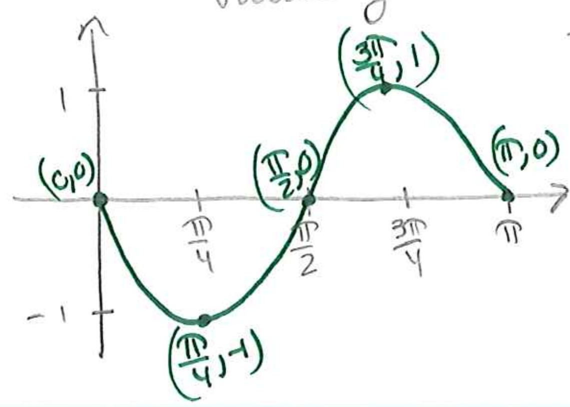
* multiply the "y" or sin values by the amplitude

Example: Graph $y = -sin(2x)$

reflection



$\omega = 2 \therefore P = \frac{2\pi}{2} = \pi \Rightarrow$ multiply "x" or radians values by 1/2

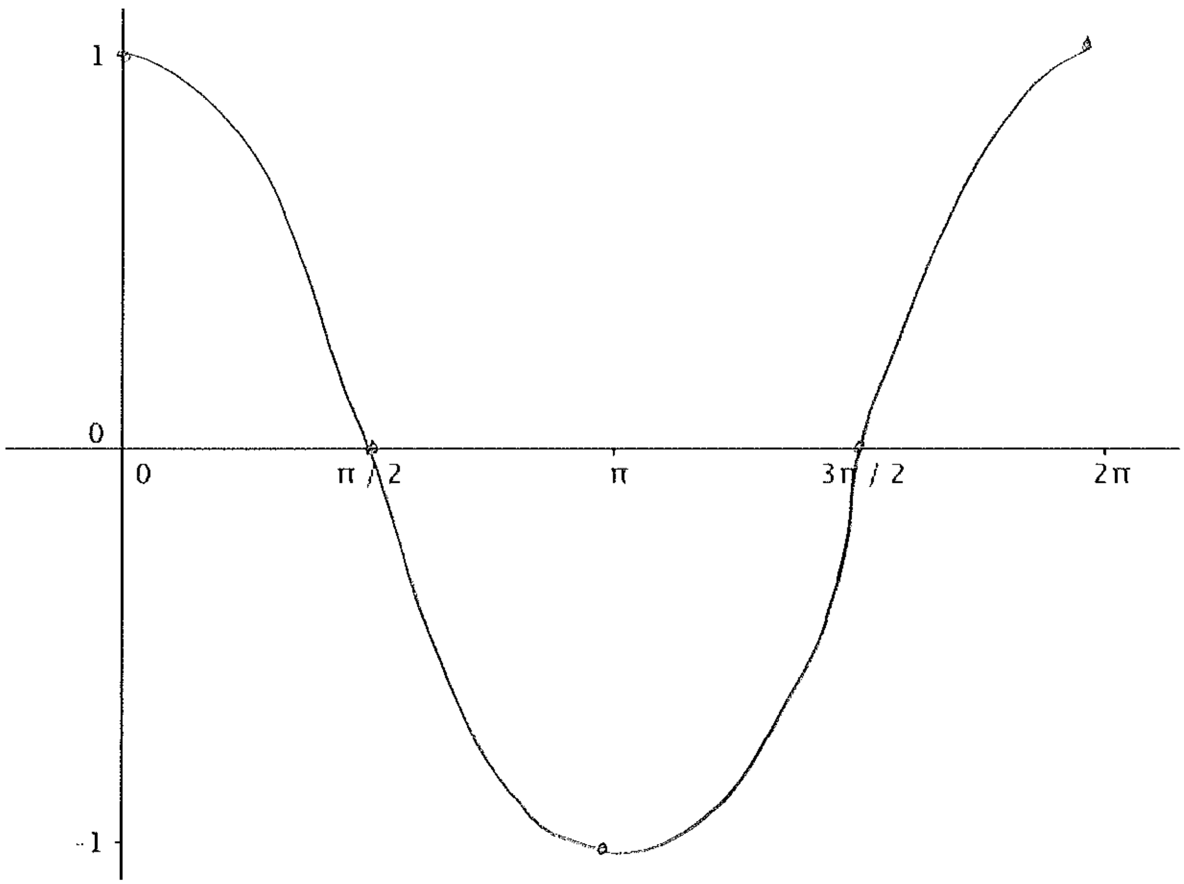


rad	rad/2	sin x
0	0	0
$\frac{\pi}{2}$	$\frac{\pi}{4}$	-1
π	$\frac{\pi}{2}$	0
$\frac{3\pi}{2}$	$\frac{3\pi}{4}$	1
2π	π	0

GRAPHING THE COSINE FUNCTION

Similar to the process of graphing sine, complete an abbreviated table below, then graph $y = \cos(x)$.

radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
cos(x)	1	0	-1	0	1

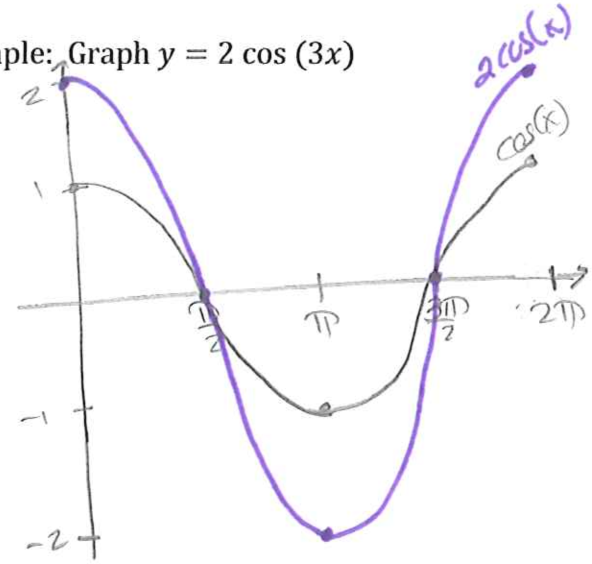


PROPERTIES OF THE COSINE FUNCTION

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The cosine function is an even function. (symmetric wrt y-axis)
4. The cosine function is periodic with a period of 2π.
5. The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the y-intercept is 1.
6. The max value is 1 and occurs at ..., -2π, 0, 2π, 4π, 6π, ...
7. The min value is -1 and occurs at ..., -π, π, 3π, 5π, ...

GRAPH FUNCTIONS OF THE FORM $y = A\cos(\omega x)$

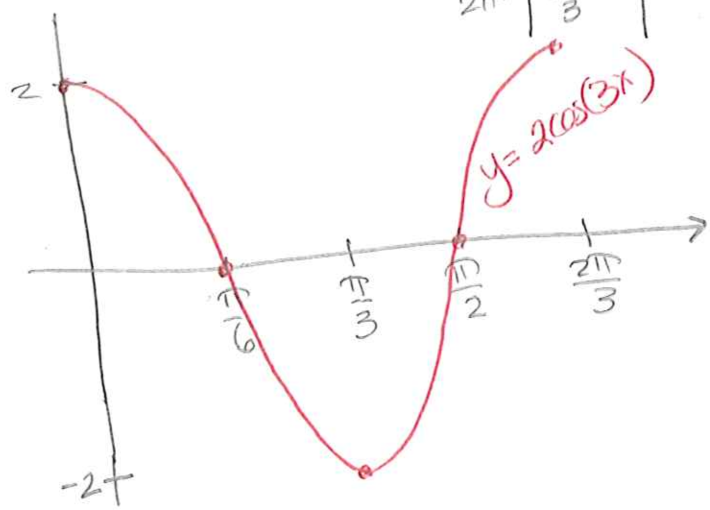
Example: Graph $y = 2\cos(3x)$



$\omega = 3$

$P = \frac{2\pi}{3}$

R	$\frac{R}{3}$	2cos(x)
0	0	2
$\frac{\pi}{2}$	$\frac{\pi}{6}$	0
π	$\frac{\pi}{3}$	-2
$\frac{3\pi}{2}$	$\frac{3\pi}{6} = \frac{\pi}{2}$	0
2π	$\frac{2\pi}{3}$	2



Pre Calc

5.4 GRAPHS OF THE SINE AND COSINE FUNCTIONS PART II

SINUSOIDAL GRAPHS

Shift the graph of y = cos(x) to the right $\frac{\pi}{2}$ units to obtain the graph of y = $\cos\left(x - \frac{\pi}{2}\right)$ (see Figure A below). Now look at the graph of y = sin(x) in Figure B. What do you notice?

$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$

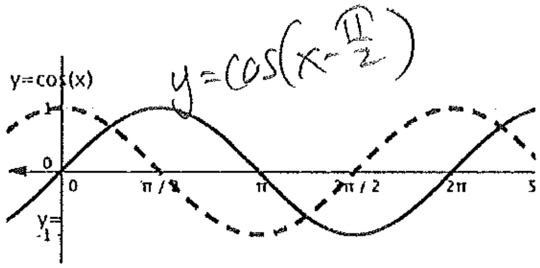


Figure A

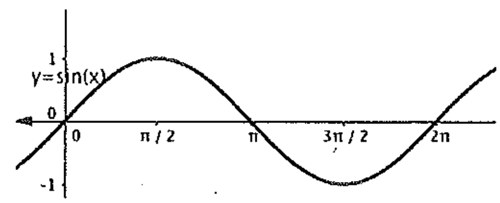


Figure B

AMPLITUDE

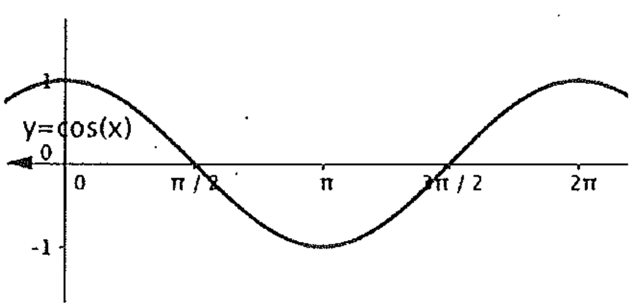
The amplitude, A, of sinusoidal functions is a vertical dilation, which is given by

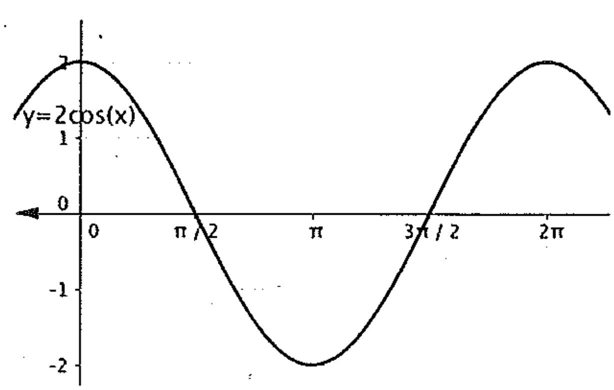
$$y = A\,sin(x) \qquad\qquad y = A\,cos(x)$$

where

$$-|A| \le A\,sin(x) \le |A| \quad and \quad -|A| \le A\,cos(x) \le |A|$$

The number |A| is called the amplitude.





Amplitude = 1

Range: $-1 \le y \le 1$

Amplitude = 2

Range: $-2 \le y \le 2$

PERIOD

The functions $y = \sin(\omega x)$ *and* $y = \cos(\omega x)$ have period $T = \frac{2\pi}{\omega}$

ω represents a horizontal dilation, either a horizontal compression or stretch by a factor of $\frac{1}{\omega}$

Therefore, the function y = sin(x) has a period, T, of $[0, 2\pi]$. However, the function $y = \sin(\omega x)$ has a period of $[0, \frac{2\pi}{\omega}]$. One period of a sinusoidal graph is a called a cycle.

Example: Find the period of y= cos(3x). $\omega = 3$

$$\left[0, \frac{2\pi}{\omega}\right] \Rightarrow \left[0, \frac{2\pi}{3}\right]$$

When graphing $y = \sin(\omega x)$ *and* $y = \cos(\omega x)$, we want ω to be positive. To graph $y = \sin(-\omega x)$, $\omega > 0$, *or* $y = \cos(-\omega x)$, $\omega > 0$, use the even-odd properties:

$$\sin(-\omega x) = -\sin(\omega x)$$
$$\cos(-\omega x) = \cos(\omega x)$$

THEOREM: If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ *and* $y = A\cos(\omega x)$ are given by

Amplitude = $\lvert A \rvert$ Period = $T = \frac{2\pi}{\omega}$

GRAPH SINUSOIDAL FXNS USING **KEY POINTS**

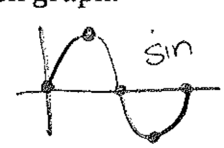
The period of sine and cosine is 2π. Dividing the period into four subintervals yields...

$$\left[0, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \pi\right], \left[\pi, \frac{3\pi}{2}\right], \left[\frac{3\pi}{2}, 2\pi\right]$$

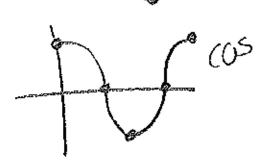
Each subinterval is of length $\frac{\pi}{2}$. These subintervals give rise to five key points on each graph.

radians sin(x)

$y = \sin(x)$: $(0,0), \left(\frac{\pi}{2}, 1\right), (\pi, 0), \left(\frac{3\pi}{2}, -1\right), (2\pi, 0)$



$y = \cos(x)$: $(0,1), \left(\frac{\pi}{2}, 0\right), (\pi, -1), \left(\frac{3\pi}{2}, 0\right), (2\pi, 1)$



HOW TO GRAPH A SINUSOIDAL FXN USING KEY POINTS

STEP 1: Determine the amplitude and period of the sinusoidal fxn.

STEP 2: Divide the interval $\left[0, \frac{2\pi}{\omega}\right]$ into four subintervals of the same length.

STEP 3: Use the endpoints of these intervals to obtain five key points on the graph.

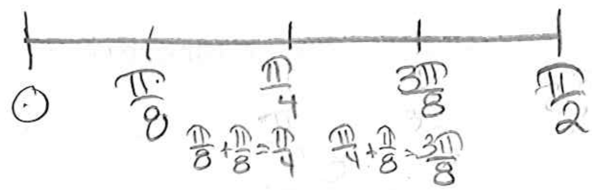
STEP 4: Plot the five key points and draw a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.

Example: Graph y = 3 sin(4x) using key points.

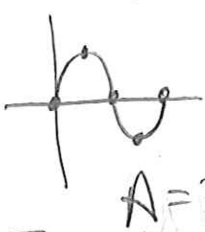
Step 1: $A = 3$ $\quad \omega = 4$ $\quad \left[0, \frac{2\pi}{\omega}\right] = \left[0, \frac{2\pi}{4}\right] =$

$T = \left[0, \frac{\pi}{2}\right]$

Step 2: $\frac{\pi}{2} \div 4 = \frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$



Step 3: Key Points

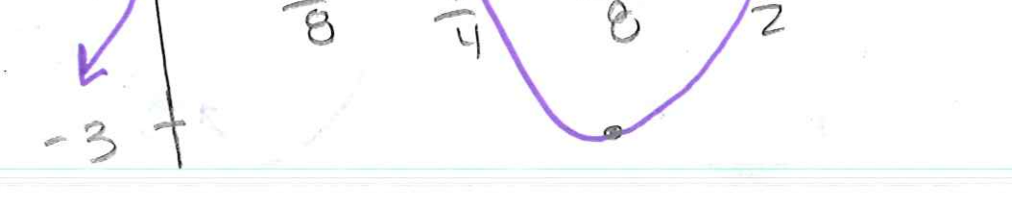


$(0, 0)\left(\frac{\pi}{8}, 1\right)\left(\frac{\pi}{4}, 0\right)\left(\frac{3\pi}{8}, -1\right)\left(\frac{\pi}{2}, 0\right)$

$A = 3$: $(0, 0), \left(\frac{\pi}{8}, 3\right), \left(\frac{\pi}{4}, 0\right)\left(\frac{3\pi}{8}, -3\right), \left(\frac{\pi}{2}, 0\right)$

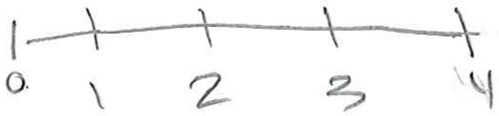
[multiply "y" by 3]

Step 4:



Example: Graph $y = 2\sin\left(-\frac{\pi}{2}x\right)$ using key points. $y = -2\sin\left(\frac{\pi}{2}x\right)$

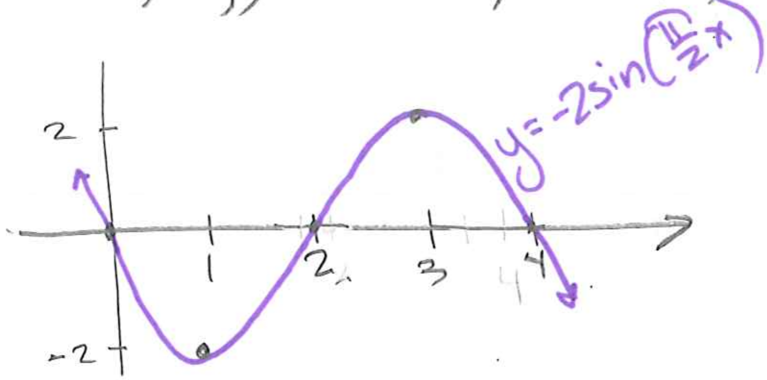
$A: -2 \; , \; w = \pi/2 \quad T = \left[0, \frac{2\pi}{(\pi/2)}\right] = [0, 4] \quad \frac{2\pi}{\frac{\pi}{2}} = \frac{2\pi}{1}\cdot\frac{2}{\pi} = 4$



$4 \div 4 = 1$

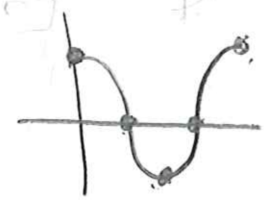
$(0,0)\;(1,1)\;(2,0)\;(3,-1)\;(4,0)$

$A=-2: (0,0), (1,-2), (2,0), (3,2), (4,0)$



Example: Graph $y = -4\cos(\pi x) - 2$ using key points.

$A = -4 \; , \; w = \pi \quad T = \left[0, \frac{2\pi}{\pi}\right] = [0, 2]$

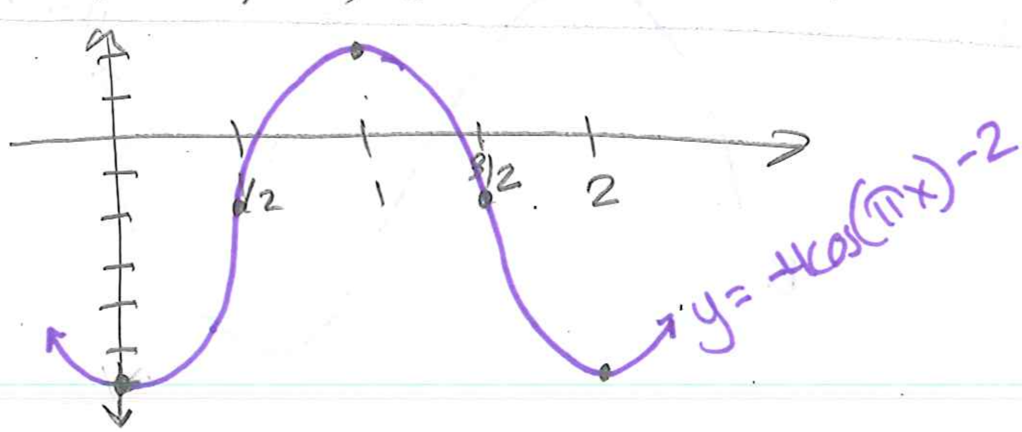


$2 \div 4 = \frac{1}{2}$

$(0,1)\;(\frac{1}{2},0)\;(1,-1)\;(3/2,0)\;(2,1)$

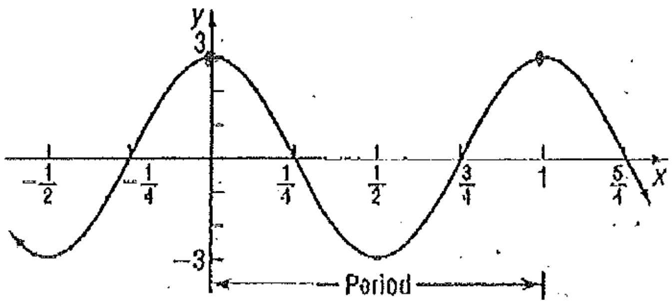
$A=-4$: $(0,-4)\;(\frac{1}{2},0)\;(1,4)\;(3/2,0)\;(2,-4)$
[mult by 4]

VS: -2: $(0,-6)\;(\frac{1}{2},-2)\;(1,2)\;(3/2,-2)\;(2,-6)$
[subt. -2]



FINDING AN EQUATION FOR A SINUSOIDAL GRAPH

Example: Find the equation for the graph shown below.



cos

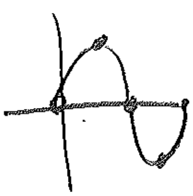
A: 3

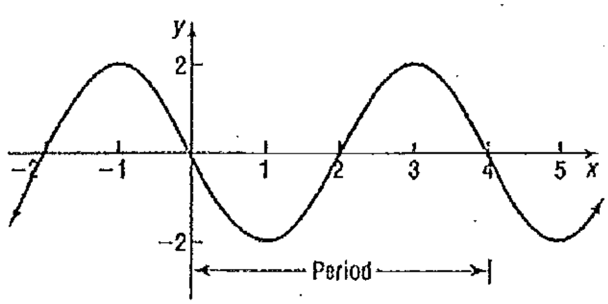
$T = 1 = \frac{2\pi}{\omega}$

$\omega = 2\pi$

$y = 3\cos(2\pi x)$

Example: Find the equation for the graph shown below.





sin

A = −2

$T = 4 = \frac{2\pi}{\omega}$

$\frac{4\omega}{4} = \frac{2\pi}{4}$

$\omega = \frac{\pi}{2}$

$y = -2\sin\left(\frac{\pi}{2}x\right)$