PreCalc

5.6 PHASE SHIFT; SINUSOIDAL CURVE FITTING NOTES

We have explored graphs of the form $y = A\sin(\omega x)$, *where* $\omega > 0$. The graph has amplitude |A| and a period $T = \frac{2\pi}{\omega}$. One cycle of this graph spans from $\left[0, \frac{2\pi}{\omega}\right]$.

Now consider the graph of

$$y = A\sin(\omega x - \phi)$$

which can also be written as

$$y = A\sin\left[\omega\left(x - \frac{\phi}{\omega}\right)\right]$$

where $\omega > 0$ and ϕ are real numbers. ϕ represents a horizontal shift. The period begins with

$$\omega x - \phi = 0 \quad or \quad x = \frac{\phi}{\omega}$$

and will end when

$$\omega x - \phi = 2\pi \quad or \quad x = \frac{\phi}{\omega} + \frac{2\pi}{\omega}$$

PHASE SHIFT VS. HORIZONTAL SHIFT

A sinusoidal function in the form $y = A\sin(\omega x - \phi) + k$, the phase shift is ϕ and $\frac{|\phi|}{2\pi}$ represents the fraction of a period that the graph has been shifted.

A sinusoidal function in the form of $y = A\sin\left[\omega\left(x - \frac{\phi}{\omega}\right)\right] + k$, the horizontal shift is $\frac{\phi}{\omega}$

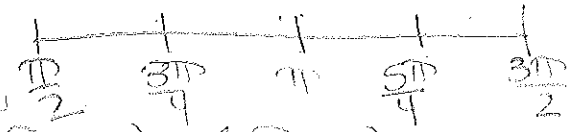
Example: Find the amplitude, period, and phase shift of $y = 3\sin(2x - \pi)$, and graph the function.

$A = 3$, $w = 2 \Rightarrow T = \frac{2\pi}{2} = \pi$, $\phi = \pi$,

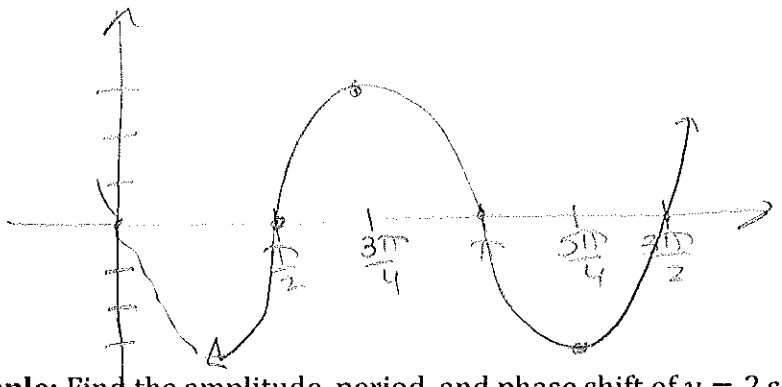
$y = 3\sin\left[2\left(x - \frac{\pi}{2}\right)\right]$ h.s is $\frac{\pi}{2}$ to right

cycle begins $x = \frac{\pi}{2}$, ends @ $\frac{\pi}{2} + \frac{2\pi}{2} = \frac{\pi}{2} + \pi = \frac{3\pi}{2} \Rightarrow \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$\frac{3\pi}{2} - \frac{\pi}{2} = \frac{2\pi}{2} = \pi \div 4 = \frac{\pi}{4}$



$\left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{4}, 3\right), (\pi, 0), \left(\frac{5\pi}{4}, -3\right), \left(\frac{3\pi}{2}, 0\right)$



Example: Find the amplitude, period, and phase shift of $y = 2\cos(4x + 3\pi) + 1$, and graph the function.

$y = 2\cos\left[4\left(x + \frac{3\pi}{4}\right)\right] + 1$ $A = 2$, $w = 4$, $\phi = 3\pi$, H.S. $-\frac{3\pi}{4}$

cycle begins: $x = -\frac{3\pi}{4}$ ends @ $-\frac{3\pi}{4} + \frac{2\pi}{4} = -\frac{\pi}{4} \Rightarrow \left[-\frac{3\pi}{4}, -\frac{\pi}{4}\right]$

$-\frac{\pi}{4} - \left(-\frac{3\pi}{4}\right) = \frac{2\pi}{2} \div 4 = \frac{\pi}{8}$

$\left(-\frac{3\pi}{4}, 1\right), \left(-\frac{5\pi}{8}, 0\right), \left(-\frac{\pi}{2}, -1\right), \left(-\frac{3\pi}{8}, 0\right), \left(-\frac{\pi}{4}, 1\right)$

$\left(-\frac{3\pi}{4}, 2\right), \left(-\frac{5\pi}{8}, 0\right), \left(-\frac{\pi}{2}, -2\right), \left(-\frac{3\pi}{8}, 0\right), \left(-\frac{\pi}{4}, 2\right)$

$\left(-\frac{3\pi}{4}, 3\right), \left(-\frac{5\pi}{8}, 1\right), \left(-\frac{\pi}{2}, -1\right), \left(-\frac{3\pi}{8}, 1\right), \left(-\frac{\pi}{4}, 3\right)$