### 6.1 Discrete Random Variables

## Learning Objectives:

1. Compute probabilities using the probability distribution of a discrete random variable.
2. Calculate and interpret the mean (expected value) of a discrete random variable.
3. Calculate and interpret the standard deviation of a discrete random variable.
4. Compute probabilities using the probability distribution of certain continuous random variables.

Vocabulary: random variable, probability distribution, discrete random variables, mean of a discrete random variable, expected value, standard deviation of an random variable, continuous random variable

What is a random variable? Give some examples.

What is a probability distribution?

What is a discrete random variable? Give some examples.

Alternate Example: How many languages?
Imagine selecting a U.S. high school student at random. Define the random variable $X=$ number of languages spoken by the randomly selected student. The table below gives the probability distribution of $X$, based on a sample of students from the U.S. Census at School database.

| Languages: | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Probability: | 0.630 | 0.295 | 0.065 | 0.008 | 0.002 |

(a) Show that the probability distribution for $X$ is legitimate.
(b) Make a histogram of the probability distribution. Describe what you see.
(c) What is the probability that a randomly selected student speaks at least 3 languages? More than 3 ?

## Alternate Example: Roulette

One wager players can make in Roulette is called a "corner bet." To make this bet, a player places his chips on the intersection of four numbered squares on the Roulette table. If one of these numbers comes up on the wheel and the player bet $\$ 1$, the player gets his $\$ 1$ back plus $\$ 8$ more. Otherwise, the casino keeps the original $\$ 1$ bet. If $X=$ net gain from a single $\$ 1$ corner bet, the possible outcomes are $x=-1$ or $x=8$. Here is the probability distribution of $X$ :

| Value: | $-\$ 1$ | $\$ 8$ |
| :--- | :--- | :--- |
| Probability: | $34 / 38$ | $4 / 38$ |

If a player were to make this $\$ 1$ bet over and over, what would be the player's average gain?

Read 350-352
How do you calculate the mean (expected value) of a discrete random variable? Is the formula on the formula sheet?

How do you interpret the mean (expected value) of a discrete random variable?

Alternate Example: Calculate and interpret the mean of the random variable $X$ in the languages example on the previous page.

Does the expected value of a random variable have to equal one of the possible values of the random variable? Should expected values be rounded?

### 6.1 Discrete \& Continuous Random Variables

Read 352-354
Suppose that $X$ is a discrete random variable with probability distribution to the right, and

| Value | $x_{1}$ | $x_{2}$ | $x_{3}$ | $\ldots$ |
| :--- | :--- | :--- | :--- | :--- |
| Probability | $p_{1}$ | $p_{2}$ | $p_{3}$ | $\cdots$ | $\mu_{x}$ is the mean of $X$.

Variance of $X$ :

## Standard Deviation of $X$ :

Are these formulas on the formula sheet?
How do you interpret the standard deviation of a discrete random variable?

Use a calculator to calculate and interpret the standard deviation of X in the languages example....

Are there any dangers to be aware of when using the calculator to find the mean and standard deviation of a discrete random variable?

Read 355-358
What is a continuous random variable? Give some examples.

Is it possible to have a shoe size $=8$ ? Is it possible to have a foot length $=8$ inches?

How many possible foot lengths are there? How can we graph the distribution of foot length?

How do we find probabilities for continuous random variables?

For a continuous random variable $X$, how is $P(X<\mathrm{a})$ related to $P(X \leq \mathrm{a})$ ?

Alternate Example: Weights of Three-Year-Old Females
The weights of three-year-old females closely follow a Normal distribution with a mean of $\mu=30.7$ pounds and a standard deviation of $\sigma=3.6$ pounds. Randomly choose one three-year-old female and call her weight $X$.
(a) Find the probability that the randomly selected three-year-old female weighs at least 30 pounds.
(b) Find the probability that a randomly selected three-year-old female weighs between 25 and 35 pounds.
(c) If $P(X<k)=0.8$, find the value of $k$.

HW page 360 (14, 18, 19, 23, 25, 27-30)

