PreCalc

6.2 The Inverse Trig Fxns (Continued)

OBJECTIVES

1. Find the exact value of expressions involving the inverse sine, cosine, and tangent fxns.
2. Define the inverse secant, cosecant, and cotangent fxns.

example:

$\sin(\cos^{-1}(\frac{1}{2}))$

let $\theta = \cos^{-1}(\frac{1}{2})$ $0 \le \theta \le \pi$

$\cos\theta = \frac{1}{2}$ $0 \le \theta \le \pi$

$\theta = \frac{\pi}{3}$

$\sin(\frac{\pi}{3}) = \boxed{\frac{\sqrt{3}}{2}}$

RECALL

$$sin\theta = \frac{y}{r} \qquad cos\theta = \frac{x}{r} \qquad tan\theta = \frac{y}{x}$$

$$csc\theta = \frac{r}{y} \qquad sec\theta = \frac{r}{x} \qquad cot\theta = \frac{x}{y}$$

When the inputs are values that are NOT familiar to the unit circle, then default to the identities above, as the radius may not equal one.

Example. Find the exact value of sin $[tan^{-1}\left(\frac{1}{2}\right)]$. *Notice ½ is not a familiar tan value on the unit circle.*

let $\theta = \tan^{-1}(\frac{1}{2})$

then $\tan\theta = \frac{1}{2}$ $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\tan\theta = \frac{1}{2} = \frac{y}{x}$ $\therefore y = 1, x = 2$

$x^2 + y^2 = r^2$

$(2)^2 + (1)^2 = r^2$

$r^2 = 5$

$r = \sqrt{5}$

If $\tan^{-1}(\frac{1}{2}) = \theta$

take sin of both sides

$\sin[\tan^{-1}(\frac{1}{2})] = \sin\theta = \frac{y}{r}$

$= \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\therefore \sin[\tan^{-1}(\frac{1}{2})] = \frac{\sqrt{5}}{5}$

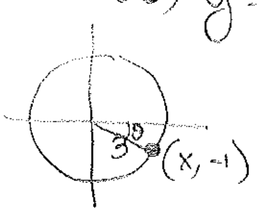
Example. Find the exact value of cos $[sin^{-1}\left(-\frac{1}{3}\right)]$.

let $\theta = \sin^{-1}(-\frac{1}{3})$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

since $(-) \Rightarrow -\frac{\pi}{2} \le \theta < 0$ (in Q IV)

then $\sin\theta = -\frac{1}{3} = \frac{y}{r}$

so; $y = -1, r = 3$



$x^2 + y^2 = r^2$

$x^2 + (-1)^2 = 3^2$

$x^2 + 1 = 9$

$x^2 = 8$

$x = 2\sqrt{2}$

If $\sin^{-1}(-\frac{1}{3}) = \theta$

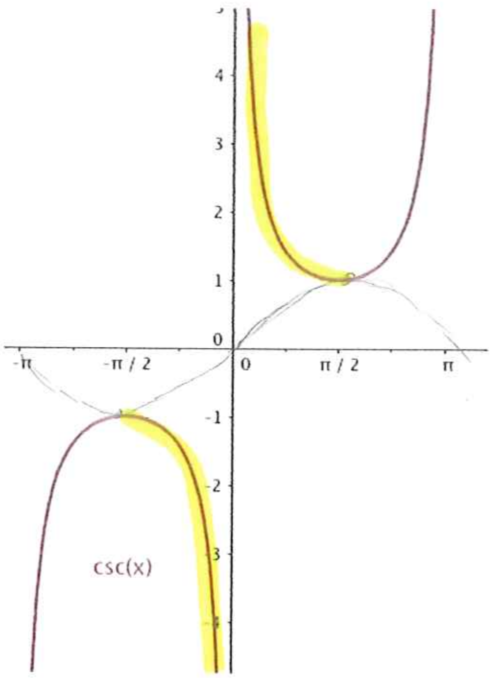
take cos of both sides

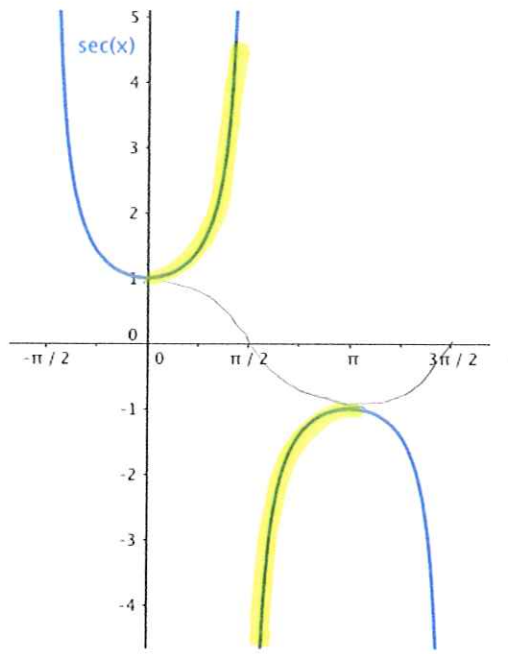
$\cos[\sin^{-1}(-\frac{1}{3})] = \cos\theta = \frac{x}{r}$

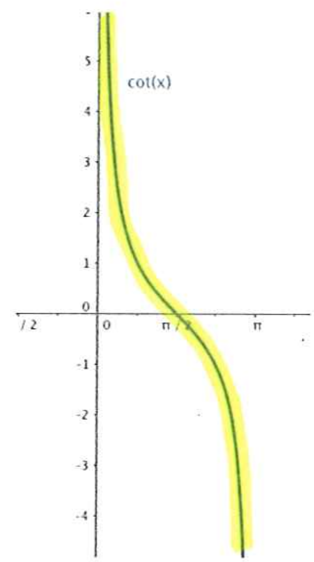
$= \boxed{\frac{2\sqrt{2}}{3}}$

DEFINE THE INVERSE SECANT, COSECANT, AND COTANGENT FXNS

Recall the graphs of secant, cosecant, and cotangent.







The inverse secant, inverse cosecant, and inverse cotangent functions are defined as follows:

$$y = sec^{-1}(x) \quad means \quad x = \sec(y)$$

$$where \quad |x| \geq 1 \quad and \quad 0 \leq y \leq \pi, \ \ y \neq \frac{\pi}{2}$$

Recall $\sec\theta = \frac{1}{\cos\theta}$

$$y = csc^{-1}(x) \quad means \quad x = \csc(y)$$

$$where \quad |x| \geq 1 \quad and \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, \ \ y \neq 0$$

Recall $\csc\theta = \frac{1}{\sin\theta}$

$$y = cot^{-1}(x) \quad means \quad x = \cot(y)$$

$$where \quad -\infty < x < \infty \quad and \quad 0 < y < \pi$$

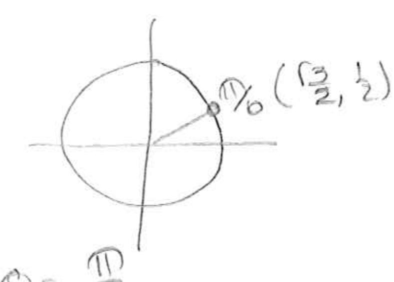
Recall $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

Example. Find the exact value of $csc^{-1}(2)$

Let $\theta = \csc^{-1}(2)$ $\quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \ \theta \neq 0$

then: $\csc\theta = 2 \Rightarrow \frac{1}{\sin\theta} = \frac{2}{1}$

$\sin\theta = \frac{1}{2}$



$\theta = \frac{\pi}{6}$