# 6.2 Transforming Random Variables

**Learning Objectives**:
1. Describe the effects of transforming a random variable by adding or subtracting a constant and multiplying or dividing by a constant.
2. Find the mean and standard deviation of the sum or difference of independent random variables.
3. Find probabilities involving the sum or difference of independent Normal random variables.

**Vocabulary:** linear transformation, independent random variables

*Read 363–369*
**Alternate Example**: El Dorado Community College
El Dorado Community College considers a student to be full-time if he or she is taking between 12 and 18 units. The number of units $X$ that a randomly selected El Dorado Community College full-time student is taking in the fall semester has the following distribution.

| Number of Units ($X$) | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|
| Probability | 0.25 | 0.10 | 0.05 | 0.30 | 0.10 | 0.05 | 0.15 |

Calculate <u>and</u> interpret the mean and standard deviation of $X$.

| *Make a histogram in your calculator:* | *To change bin widths*: |
|---|---|
| 1. Press 2nd Y = (Stat Plot)<br>2. Turn on a plot, select histogram<br>3. Xlist = L1 and Freq = L2<br>4. Press Zoom → 9. ZoomStat | 1. Press WINDOW<br>2. Change Xmin, Xmax, Xscl<br>* Use 12, 18, 1 for this example |

At El Dorado Community College, the tuition for full-time students is $50 per unit. So, if $T$ = tuition charge for a randomly selected full-time student, $T = 50X$. Here's the probability distribution for $T$:

| Tuition Charge ($T$) | 600 | 650 | 700 | 750 | 800 | 850 | 900 |
|---|---|---|---|---|---|---|---|
| Probability | 0.25 | 0.10 | 0.05 | 0.30 | 0.10 | 0.05 | 0.15 |

Calculate the mean and standard deviation of $T$.

What is the effect of multiplying or dividing a random variable by a constant?

In addition to tuition charges, each full-time student at El Dorado Community College is assessed student *PERIOD 4:* fees of \$100 per semester. If $C$ = overall cost for a randomly selected full-time student, $C = 100 + T$. Here is the probability distribution for $C$:

| Overall Cost ($C$) | 700 | 750 | 800 | 850 | 900 | 950 | 1000 |
|---|---|---|---|---|---|---|---|
| Probability | 0.25 | 0.10 | 0.05 | 0.30 | 0.10 | 0.05 | 0.15 |

Calculate the mean and standard deviation of $C$.

What is the effect of adding (or subtracting) a constant to a random variable?

**LINEAR TRANSFORMATIONS** – for any random variable X, a linear transformation in the format of

____________________________ , where a = ______________________________________________________ and

b = __________________________________________________ describes the sequence of transformations.

How are the shape, center, and spread affected by a linear transformation?

**Alternate Example:** *Scaling a Test*

In a large introductory statistics class, the distribution of $X$ = raw scores on a test was approximately normally distributed with a mean of 17.2 and a standard deviation of 3.8. The professor decides to scale the scores by multiplying the raw scores by 4 and adding 10.

(a) Define the variable $Y$ to be the scaled score of a randomly selected student from this class. Find the mean and standard deviation of $Y$.

(b) What is the probability that a randomly selected student has a scaled test score of at least 90?

**HW page 382 (35, 37, 39, 40, 41, 43, 45)**

## 6.2 Combining Random Variables

*Read pages 369–381*

**Mean of the Sum of Random Variables (RVs):**
For any RVs, X and Y, if T = X + Y, then the expected value of T is

**In general, the mean of the sum of several random variables is the sum of their means.

**Example**: *Touring Adventures*
Siblings Pete and Erin give half-day trips as tour guides for their cities, San Francisco and Charleston respectively. Below are the probability distributions for each tour business. Let X = the number of passengers on a randomly selected half-day trip with Pete and let Y = the number of passengers on a randomly selected half day-trip with Erin.

| **X** | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| **P(x)** | 0.15 | 0.25 | 0.35 | 0.20 | 0.05 |

| **Y** | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| **P(y)** | 0.3 | 0.4 | 0.2 | 0.1 |

If T is the total passengers that Pete and Erin have on their tours on a randomly selected day, what is the average total?

How much variability is there in the total number of passengers who go on Pete's and Erin's tours on a randomly chosen day?

If knowing whether any event involving X alone has occurred tells us nothing about the occurrence of any event involving Y alone, and vice versa, then X and Y are ____________________________ **random variables**.

For any two *independent* RVs X and Y, if T = X + Y, then the variance of T is

In general, the variance of the sum of several *independent* RVs is the sum of their variances.

**Alternate Example**: Suppose that a certain variety of apples have weights that are approximately Normally distributed with a mean of 9 ounces and a standard deviation of 1.5 ounces. If bags of apples are filled by randomly selecting 12 apples, what is the probability that the sum of the 12 apples is less than 100 ounces?

**Alternate Example**: Let $B$ = the amount spent on books in the fall semester for a randomly selected full-time student at El Dorado Community College. Suppose that $\mu_B = 153$ and $\sigma_B = 32$. Recall from earlier that $C$ = overall cost for tuition and fees for a randomly selected full-time student at El Dorado Community College and $\mu_C$ = 832.50 and $\sigma_C$ = 103. Find the mean and standard deviation of the cost of tuition, fees and books ($C + B$) for a randomly selected full-time student at El Dorado Community College.

**HW page 384 (47, 51, 57, 58, 59, 61, 63, 65, 66)**