6.3 Binomial Distributions

Learning Objectives:

1. Determine whether the conditions for using a binomial random variable are met.
2. Compute and interpret probabilities involving binomial distributions.
3. Calculate the mean and standard deviation of a binomial random variable, and interpret these values in context.
4. Find probabilities involving geometric random variables.
5. When appropriate, use the Normal approximation to the binomial distribution to calculate probabilities.

Vocabulary: binomial setting, binomial random variable, binomial distribution, binomial coefficient, factorial, 10% condition, geometric setting, geometric random variable, geometric distribution, geometric probability

Activity: *M & Ms*

I. Binomial Distributions

1. The objective is to model the binomial distribution using blue M&Ms as the sampling object.
2. Select a sample of 10 M&Ms (without looking) from the paper bag. Let **X** = the number of blue M&Ms.
3. Put the M&Ms back and repeat 10 times (10 trials), keeping track of the number of blue M&Ms. Record in the table below.

Individual Data										
Trial	1	2	3	4	5	6	7	8	9	10
X										

4. Fill in group data (each group = 4 students). What was the smallest X value observed in the group? Largest X value? Your table should show all possible values of X between these values.
5. Count the total number of tallies to find P(X)s (*i.e. groups of 4 = 40 trials, so if tally for X = 1 is 5, P(1) = 5/40*).
6. Draw a histogram (use proportions) for the results for your group.

Group Data								
X								
Tally								
P(X)								

Reflection questions:

1. Describe the shape of the histogram.

2. We only kept track of blue M&Ms (not green, red, etc), so what TWO OUTCOMES were possible?

3. Explain the purpose of returning the 10 M&Ms to the bag.

II. Geometric Distribution:

1. Select an M&M (without looking) from the paper bag. If the M&M is blue, the trial is finished. If the M&M is not blue, return the M&M to the bag and continue drawing M&Ms one at a time until a blue is drawn. Let **X** = the number of M&Ms until a blue is drawn.
2. Repeat the process until 10 trials are completed and record below.

Individual Data												
Trial	1	2	3	4	5	6	7	8	9	10	11	12
X												

3. Fill in the group data. What was the smallest X value observed in the group? Largest X value? Your table should show all possible values of X between these values.
4. Count the total number of tallies to find P(X)s (ie. group of 4 =40 total trials, so if tall for X = 1 is 5, P(1) = 5/40)
5. Draw a histogram (using proportions) for the results for your group.

Group Data															
X															
Tally															
P(X)															

Reflection questions:

1. Describe the shape of the histogram.

2. We only kept track of blue M&Ms (not green, red, etc), so what TWO OUTCOMES were possible?

3. Explain the purpose of returning the 10 M&Ms to the bag.

What are similarities/differences between the Binomial and Geometric settings?

Setting	Binomial	Geometric
Binary: The number of outcomes of each trial is _______.		
Independent: Trials must be independent of each other. Knowing the result of one trial must not tell us anything about the result of any other trial.		
Number of Trials:		
Success: The probability of success, *p*, on each trial is the same.		

Acronym for settings: **BINS**.

Determine whether the random variables below have a binomial distribution, geometric distribution, or neither. Justify your answer.

(a) Roll a fair die 10 times and let X = the number of sixes.

(b) Shoot a basketball 20 times from various distances on the court. Let Y = number of shots made.

(c) Shuffle a deck of cards and choose a random card. Replace the card and repeat this process until you get an ace.

(d) Observe the next 100 cars that go by and let C = color.

Read 386–389

What are the conditions for a binomial setting? ***Remember to check the BINS!!***

- __________________? The possible outcomes of each trial can be classified as "success" or "failure."
- ________________________? Trials must be independent; that is, knowing the result of one trial must not tell us anything about the result of any other trial.
- _____________________? The number of trials *n* of the chance process must be fixed in advanced.
- _________________________________? There is the same probability *p* of success on each trial.

The count *X* of successes in a binomial setting is a _________________________________. The probability distribution of *X* is a _______________________________ with parameters *n* and *p*, where *n* is the _________________________________ and *p* is the ____ on any one trial. The possible values of *X* are the whole numbers (discrete values) from ______________________.

Alternate Example: *Rolling Sixes*
In many games involving dice, rolling a 6 is desirable. The probability of rolling a six when rolling a fair die is 1/6. If X = the number of sixes in 4 rolls of a fair die, then X is binomial with $n = 4$ and $p = 1/6$.

What is $P(X = 0)$? That is, what is the probability that all 4 rolls are *not* sixes?

What is $P(X = 1)$?

What about $P(X = 2)$, $P(X = 3)$, $P(X = 4)$?

In general, how can we calculate binomial probabilities? Is the formula on the formula sheet?

Alternate Example: *Roulette*
In Roulette, 18 of the 38 spaces on the wheel are black. Suppose you observe the next 10 spins of a roulette wheel.

(a) What is the probability that exactly 4 of the spins land on black?

(b) What is the probability that at least 8 of the spins land on black?

Technology: Binomial Probabilty TI-84 (p. 394)

$\boxed{2^{nd}}$ → $\boxed{\text{VARS}}$

binompdf(n, p, k) computes $P(X = k)$

binomcdf(n, p, k) computes $P(X \leq k)$

Is it OK to use the binompdf and binomcdf commands on the AP exam?

What is the most common mistake students make on binomial distribution questions?

HW page 410 (69–79 odd)

Read 397-400
How can you calculate the mean and SD of a binomial distribution? Are these on the formula sheet?

Alternate Example: *Roulette*
Let X = the number of the next 10 spins of a roulette wheel that land on black.
(a) Calculate and interpret the mean and standard deviation of X.

(b) How often will the number of spins that land on black be within one standard deviation of the mean?

Read 401–402

When is it OK to use the binomial distribution when sampling without replacement? Why is this an issue?

<u>**10% Condition**</u>
When taking an SRS of size n from a population of size, N, we can use a binomial distribution to model the count of successes in the sample as long as…

Alternate Example: In the NASCAR Cards and Cereal Boxes example from Section 5.1, we read about a cereal company that put one of 5 different cards into each box of cereal. Each card featured a different driver: Jeff Gordon, Dale Earnhardt, Jr., Tony Stewart, Danica Patrick, or Jimmie Johnson. Suppose that the company printed 20,000 of each card, so there were 100,000 total boxes of cereal with a card inside. If a person bought 6 boxes at random, what is the probability of getting 2 Danica Patrick cards?

HW p. 411 (72–78 even, 81–89 odd)

6.3 The Geometric Distribution

Read 404–406
A ______________________________arises when we perform independent trials of the same chance process with and record the number of trials it takes to get one success. One each trial, the probability, *p,* of success must be the same.

The number of trials Y that it takes to get a success in a geometric setting is a __________________________ _________________________. The probability distribution of Y is a geometric distribution with parameter:

Alternate Example: *Monopoly*
In the board game Monopoly, one way to get out of jail is to roll doubles. Suppose that a player has to stay in jail until he or she rolls doubles. The probability of rolling doubles is 1/6.
(a) Explain why this is a geometric setting.

(b) Define the geometric random variable and state its distribution.

(c) Find the probability that it takes exactly three rolls to get out of jail. Exactly 4 rolls? 100 rolls?

In general, how can you calculate geometric probabilities? Is this formula on the formula sheet?

In general, how do you calculate the **mean** of a geometric distribution? Is the formula on the formula sheet?

On average, how many rolls should it take to escape jail in Monopoly?

What is the probability it takes longer than average to escape jail? What does this probability tell you about the shape of the distribution?

HW page 412 (93, 95, 97, 98, 101–103)