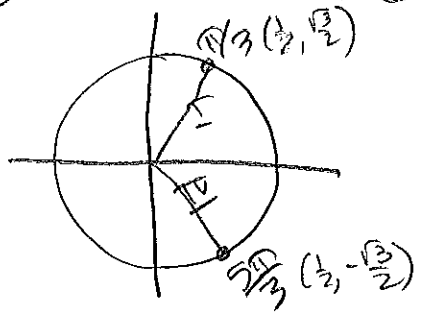
4/20/17

6.3 Part II

Finding all solns

Recall sine + cosine have a period of 2π
tangent has a period of π

① Solve $\cos\theta = \frac{1}{2}$ [find all solns (= solutions) and list 6 solns (= solutions)]



$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ for $0 \le \theta \le 2\pi$

$\left\{\frac{\pi}{3} + 2\pi k, \frac{5\pi}{3} + 2\pi k\right\}$ "all solns"
where k is an integer

$k = 1 : \frac{\pi}{3} + 2\pi(1) = \frac{\pi}{3} + \frac{2\pi}{1}\cdot\frac{3}{3} = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$

$k = -1 : \frac{\pi}{3} + 2\pi(-1) = \frac{\pi}{3} - \frac{6\pi}{3} = \frac{-5\pi}{3}$

$k = 1 : \frac{5\pi}{3} + 2\pi(1) = \frac{5\pi}{3} + \frac{6\pi}{3} = \frac{11\pi}{3}$

$k = 2 : \frac{5\pi}{3} + 2\pi(2) = \frac{5\pi}{3} + \frac{12\pi}{3} = \frac{17\pi}{3}$

$$\left\{\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, -\frac{5\pi}{3}, \frac{11\pi}{3}, \frac{17\pi}{3}\right\}$$

(2) Solve $\sin(2\theta) = \frac{1}{2}$, $0 \le \theta < 2\pi$

$\sin\theta = \frac{1}{2}$ $\quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

[period of $\sin(2\theta)$ is $\frac{2\pi}{2} = \pi$]

So $\frac{2\theta = \frac{\pi}{6} + 2\pi k}{2}$ and $\frac{2\theta = \frac{5\pi}{6} + 2\pi k}{2}$

$\theta = \frac{\pi}{12} + \pi k$ and $\theta = \frac{5\pi}{12} + \pi k$

all solns $\left\{\frac{\pi}{12} + \pi k, \frac{5\pi}{12} + \pi k\right\}$

$k=1: \frac{\pi}{12} + \pi k = \frac{\pi}{12} + \pi(1) = \frac{\pi}{12} + \frac{12\pi}{12} = \frac{13\pi}{12}$

$k=2: \frac{\pi}{12} + \pi(2) = \frac{\pi}{12} + \frac{2\pi}{1}\cdot\frac{12}{12} = \frac{\pi}{12} + \frac{24\pi}{12} = \frac{25\pi}{12}$

$k=-1: \frac{5\pi}{12} + \pi(-1) = \frac{-\pi}{12}$ . . .

$$\left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{25\pi}{12} \ldots\right\}$$

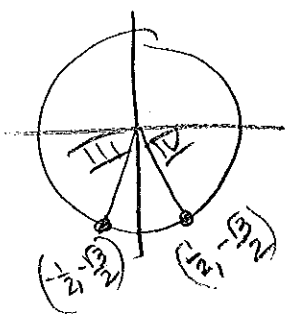
Solving Linear Trig Eqns

Solve $2\sin\theta + \sqrt{3} = 0$, $0 \le \theta < 2\pi$

$-\sqrt{3} \quad -\sqrt{3}$

$\frac{2\sin\theta}{2} = \frac{-\sqrt{3}}{2}$ $\quad \theta = \left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

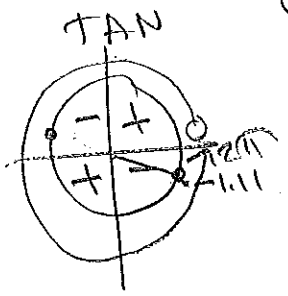
$\sin\theta = -\frac{\sqrt{3}}{2}$



Solve Trig Eqn w/ a calc

Solve $\tan\theta = -2$, $0 \le \theta < 2\pi$ [Express soln in radians and round to 2 decim.]

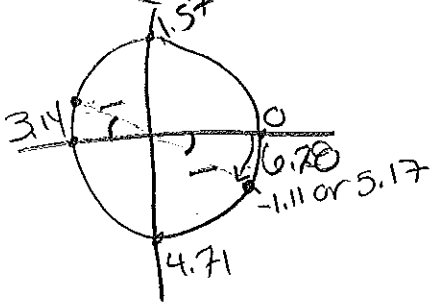
$\theta = \tan^{-1}(-2) \approx -1.11$



$-1.11 + \pi(2) = 5.17$ ($k=2$)

$-1.11 + \pi = 2.03$

$\{2.03, 5.17\}$



Solve Trig Eqn Quadratic in form

Solve $2\sin^2\theta - 3\sin\theta + 1 = 0$ $\quad 0 \le \theta < 2\pi$

Let $x = \sin\theta$

$2x^2 - 3x + 1 = 0$

a·c: 2; $-2, -1$

$2x^2 - 2x - 1x + 1 = 0$

GCF: $2x$ $\quad$ GCF: -1

$2x(x-1) - 1(x-1) = 0$

$2x[\square] - 1[\square]$

$[\square](2x-1)$

$(2x-1)(x-1) = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

[ZPP] $2\sin\theta - 1 = 0$ $\quad \sin\theta - 1 = 0$

$-3x = -2x - 1x$

$2\sin\theta - 1 = 0$

$+1 \quad +1$

$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin\theta - 1 = 0$

$+1 \quad +1$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

$\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$

Solving Trig Eqns using Identities

Solve $3\cos\theta + 3 = 2\sin^2\theta \quad 0 \le \theta < 2\pi$

$\sin^2\theta + \cos^2\theta = 1$

$-\cos^2\theta \quad -\cos^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$3\cos\theta + 3 = 2[1 - \cos^2\theta]$

$3\cos\theta + 3 = 2 - 2\cos^2\theta$

$+2\cos^2\theta \quad -2 \quad -2 \quad +2\cos^2\theta$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

Let $x = \cos\theta$

$2x^2 + 3x + 1 = 0$

$3x = 2x + 1x$

3

2, 1

$2x^2 + 2x + 1x + 1 = 0$

GCF = 2x GCF = 1

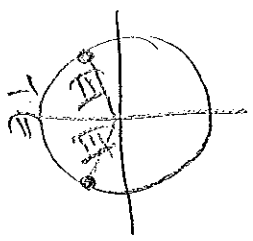
$2x(x+1) + 1(x+1) = 0$

$(2x+1)(x+1) = 0$

$2x + 1 = 0 \quad x + 1 = 0$

$2\cos\theta + 1 = 0 \quad \cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2} \quad \cos\theta = -1$



$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$

$\left\{\frac{2\pi}{3}, \frac{4\pi}{3}, \pi\right\}$