PreCalc

6.4 TRIGONOMETRIC IDENTITIES NOTES

Objectives:

1. Use Algebra to Simplify Trigonometric Expressions.
2. Establish Identities.

Two functions f and g are identically equal if $f(x) = g(x)$.

Such an equation is referred to as an identity. An equation that is not an identity is called a conditional equation.

Examples of Identities

$$(x+1)^2 = x^2 + 2x + 1$$

$$cos^2(\theta) + sin^2(\theta) = 1$$

$$\csc(x) = \frac{1}{\sin(x)}$$

Examples of conditional equations.

$2x + 5 = 0$ - equal only if $x = -5/2$

$\sin(x) = 0$ - equal only if $x = \pi, 0$

$\sin(x) = \cos(x)$

equal only if $x = \frac{\pi}{4}, \frac{3\pi}{4}$

SUMMARY OF TRIG IDENTITIES

Quotient Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Reciprocal Identities

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}$$

Pythagorean Identities

$$sin^2(\theta) + cos^2(\theta) = 1 \qquad tan^2(\theta) + 1 = sec^2(\theta) \qquad cot^2(\theta) + 1 = csc^2(\theta)$$

Even-Odd Identities

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = cos(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

$$\csc(-\theta) = -\csc(\theta) \qquad \sec(-\theta) = sec(\theta) \qquad \cot(-\theta) = -\cot(\theta)$$

USING ALGEBRAIC TECHNIQUES TO SIMPLIFY TRIG EXPRESSIONS

Examples:

a) Simplify $\frac{\cot(\theta)}{\csc(\theta)}$ by rewriting each trig fxn in terms of sine and cosine fxns.

b) Show that $\frac{\cos(\theta)}{1+\sin(\theta)} = \frac{1-\sin(\theta)}{\cos(\theta)}$ by multiplying the numerator and denominator by $1-\sin(\theta)$.

c) Simplify $\frac{1+\sin(x)}{\sin(x)} + \frac{\cot(x)-\cos(x)}{\cos(x)}$ by rewriting the expression over a common denominator.

d) Simplify $\frac{\sin^2(\theta)-1}{\tan(\theta)\sin(\theta)-\tan(\theta)}$ by factoring.

C.D = $\sin(x)\cos(x)$

a) $\frac{\cot\theta}{\csc\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$= \frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1}$

$= \cos\theta$

b) $\frac{\cos\theta}{1+\sin\theta} \cdot \frac{1-\sin\theta}{1-\sin\theta}$

$= \frac{\cos\theta(1-\sin\theta)}{1+\cancel{\sin\theta}-\cancel{\sin\theta}-\sin^2\theta}$

$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$

$= \frac{1-\sin\theta}{\cos\theta}$ ✓

c) $\frac{1+\sin(x)}{\sin(x)} \cdot \frac{\cos x}{\cos x} + \frac{\cot(x)-\cos(x)}{\cos(x)} \cdot \frac{\sin x}{\sin x}$

$= \frac{\cos x + \cancel{\cos x \sin x} + \cot(x)\sin(x) - \cancel{\cos(x)\sin(x)}}{\cos(x)\sin(x)}$

$= \frac{\cos(x) + \frac{\cos(x)}{\cancel{\sin(x)}} \cdot \cancel{\sin(x)}}{\cos(x)\sin(x)}$

$= \frac{2\cancel{\cos(x)}}{\cancel{\cos(x)}\sin(x)} = \frac{2}{\sin(x)}$

$x^2-1=(x+1)(x-1)$

d) $\frac{\sin^2\theta-1}{\tan(\theta)\sin\theta-\tan\theta} = \frac{(\sin\theta+1)\cancel{(\sin\theta-1)}}{\tan\theta\cancel{(\sin\theta-1)}}$

$= \frac{\sin\theta+1}{\tan\theta}$

ESTABLISHING AN IDENTITY

work w/ more complex side

Examples: Establish the following identities.

a) $\csc(x)\tan(x) = \sec(x)$

b) $sin^2(-\theta) + cos^2(-\theta) = 1$

c) $\frac{sin^2(-\theta) - cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos(\theta) - \sin(\theta)$

d) $\frac{1+\tan(x)}{1+\cot(x)} = \tan(x)$

e) $\frac{\sin(x)}{1+\cos(x)} + \frac{1+\cos(x)}{\sin(x)} = 2\csc(x)$

a) $\csc(x)\tan(x) = \sec(x)$

$= \frac{1}{\sin(x)} \cdot \frac{\sin(x)}{\cos(x)}$

$= \frac{1}{\cos(x)}$

$= \sec(x)$ ✓

b) $\sin^2(-\theta) + \cos^2(-\theta)$

$= [\sin(-\theta)]^2 + [\cos(-\theta)]^2$

$= [-\sin(\theta)]^2 + [\cos(\theta)]^2$

$= \sin^2\theta + \cos^2\theta$

1

c) $\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)}$

$= \frac{(-\sin\theta)^2 - (\cos\theta)^2}{-\sin(\theta) - \cos(\theta)}$

$= \frac{\sin^2\theta - \cos^2\theta}{-1(\sin(\theta) + \cos(\theta))}$

$= \frac{\cancel{(\sin\theta + \cos\theta)}(\sin\theta - \cos\theta)}{-1\cancel{(\sin\theta + \cos\theta)}}$

$= -(\sin\theta - \cos\theta)$

$= \cos\theta - \sin\theta$

d) $\frac{1+\tan(x)}{1+\cot(x)} = \frac{1+\tan(x)}{1+\frac{1}{\tan(x)}}$ $\frac{\tan x}{\tan x}$

$= \frac{1+\tan(x)}{\frac{\tan x}{\tan x} + \frac{1}{\tan x}}$

$= \frac{1+\tan(x)}{\frac{\tan(x)+1}{\tan x}} = \frac{\cancel{1+\tan x}}{1} \cdot \frac{\tan x}{\cancel{1+\tan x}}$

$= \tan x$ ✓

e) $\frac{\sin x}{\sin x} \cdot \frac{\sin(x)}{1+\cos(x)} + \frac{1+\cos(x)}{\sin(x)} \cdot \frac{1+\cos x}{1+\cos x}$ CD $\cdot (1+\cos x)(\sin x)$

$= \frac{\sin^2 x + 1 + 2\cos(x) + \cos^2 x}{\sin x(1+\cos x)}$

$= \frac{1 + 1 + 2\cos(x)}{\sin x(1+\cos x)} = \frac{2 + 2\cos x}{\sin(x)(1+\cos x)}$

$= \frac{2\cancel{(1+\cos x)}}{\sin(x)\cancel{(1+\cos x)}} = \frac{2}{\sin x} = 2 \cdot \frac{1}{\sin x}$

$= 2\csc x$