PreCalc

TH 5/2/17

6.5 Sum & Difference Formulas NOTES

Objectives

1. Use Sum & Difference Formulas to find exact values.
2. Use Sum & Difference Formulas to establish identities.

Sum & Difference formulas involve the sum or difference of two angles, such as $\cos(\alpha+\beta)$ & $\cos(\alpha-\beta)$.

Sum & Difference Formulas

$\theta = \alpha + \beta$

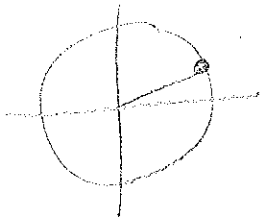
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$



Example. Use the Sum Formula to find an exact value of cos (75°)

$$\cos(75^\circ) = \cos(45^\circ + 30^\circ) = \cos45\cdot\cos30 - \sin45\sin30$$

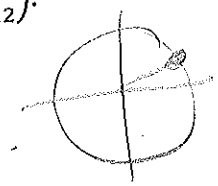
$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4} \text{ or } \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$$

Example. Use the Difference Formula to find the exact value of $\cos\left(\frac{\pi}{12}\right)$.

$\frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$



$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$ or $\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$

Example. Use the Sum Formula to find the exact value of $\sin\left(\frac{7\pi}{12}\right)$.

$\frac{7\pi}{12} = \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

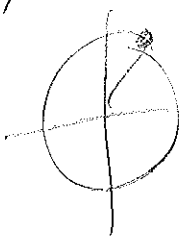
$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$ or $\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$

Example. Use the ~~Sum~~ Difference Formula to find an exact value of $\sin 80° \cos 20° - \cos 80° \sin 20°$

$\sin(80-20)$

$\sin(60)$

$\frac{\sqrt{3}}{2}$



Assign 6.5: 5, 6, 8, 10, 11-17 odd