

With this formula, the solution to Example 7(c) can be obtained as follows:

$$\cos \alpha = -\frac{3}{5} \quad \pi < \alpha < \frac{3\pi}{2}$$

$$\sin \alpha = -\sqrt{1 - \cos^2 \alpha} = -\sqrt{1 - \frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

Then, by equation (11),

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{1 - \left(-\frac{3}{5}\right)}{-\frac{4}{5}} = \frac{\frac{8}{5}}{-\frac{4}{5}} = -2$$

6.6 Assess Your Understanding

Concepts and Vocabulary

1. $\cos(2\theta) = \cos^2 \theta - \underline{\quad} = \underline{\quad} - 1 = 1 - \underline{\quad}$.

2. $\sin^2 \frac{\theta}{2} = \frac{\underline{\quad}}{2}$

3. $\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\underline{\quad}}$

4. *True or False* $\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

5. *True or False* $\sin(2\theta)$ has two equivalent forms:

$$2 \sin \theta \cos \theta \quad \text{and} \quad \sin^2 \theta - \cos^2 \theta$$

6. *True or False* $\tan(2\theta) + \tan(2\theta) = \tan(4\theta)$

Skill Building

In Problems 7–18, use the information given about the angle θ , $0 \leq \theta < 2\pi$, to find the exact value of:

(a) $\sin(2\theta)$

(b) $\cos(2\theta)$

(c) $\sin \frac{\theta}{2}$

(d) $\cos \frac{\theta}{2}$

(e) $\tan(2\theta)$

(f) $\tan \frac{\theta}{2}$

7. $\sin \theta = \frac{3}{5}$, $0 < \theta < \frac{\pi}{2}$

8. $\cos \theta = \frac{3}{5}$, $0 < \theta < \frac{\pi}{2}$

9. $\tan \theta = \frac{4}{3}$, $\pi < \theta < \frac{3\pi}{2}$

10. $\tan \theta = \frac{1}{2}$, $\pi < \theta < \frac{3\pi}{2}$

11. $\cos \theta = -\frac{\sqrt{6}}{3}$, $\frac{\pi}{2} < \theta < \pi$

12. $\sin \theta = -\frac{\sqrt{3}}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$

13. $\sec \theta = 3$, $\sin \theta > 0$

14. $\csc \theta = -\sqrt{5}$, $\cos \theta < 0$

15. $\cot \theta = -2$, $\sec \theta < 0$

16. $\sec \theta = 2$, $\csc \theta < 0$

17. $\tan \theta = -3$, $\sin \theta < 0$

18. $\cot \theta = 3$, $\cos \theta < 0$

In Problems 19–28, use the Half-angle Formulas to find the exact value of each expression.

19. $\sin 22.5^\circ$

20. $\cos 22.5^\circ$

21. $\tan \frac{7\pi}{8}$

22. $\tan \frac{9\pi}{8}$

23. $\cos 165^\circ$

24. $\sin 195^\circ$

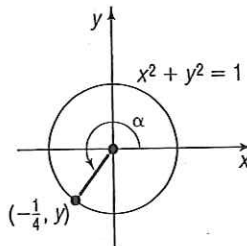
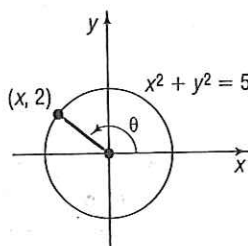
25. $\sec \frac{15\pi}{8}$

26. $\csc \frac{7\pi}{8}$

27. $\sin\left(-\frac{\pi}{8}\right)$

28. $\cos\left(-\frac{3\pi}{8}\right)$

In Problems 29–40, use the figures to evaluate each function given that $f(x) = \sin x$, $g(x) = \cos x$, and $h(x) = \tan x$.



29. $f(2\theta)$ 30. $g(2\theta)$ 31. $g\left(\frac{\theta}{2}\right)$ 32. $f\left(\frac{\theta}{2}\right)$
 33. $h(2\theta)$ 34. $h\left(\frac{\theta}{2}\right)$ 35. $g(2\alpha)$ 36. $f(2\alpha)$
 37. $f\left(\frac{\alpha}{2}\right)$ 38. $g\left(\frac{\alpha}{2}\right)$ 39. $h\left(\frac{\alpha}{2}\right)$ 40. $h(2\alpha)$

41. Show that $\sin^4 \theta = \frac{3}{8} - \frac{1}{2} \cos(2\theta) + \frac{1}{8} \cos(4\theta)$.
 42. Show that $\sin(4\theta) = (\cos \theta)(4 \sin \theta - 8 \sin^3 \theta)$.
 43. Develop a formula for $\cos(3\theta)$ as a third-degree polynomial in the variable $\cos \theta$.
 44. Develop a formula for $\cos(4\theta)$ as a fourth-degree polynomial in the variable $\cos \theta$.
 45. Find an expression for $\sin(5\theta)$ as a fifth-degree polynomial in the variable $\sin \theta$.
 46. Find an expression for $\cos(5\theta)$ as a fifth-degree polynomial in the variable $\cos \theta$.

In Problems 47–68, establish each identity.

47. $\cos^4 \theta - \sin^4 \theta = \cos(2\theta)$ 48. $\frac{\cot \theta - \tan \theta}{\cot \theta + \tan \theta} = \cos(2\theta)$ 49. $\cot(2\theta) = \frac{\cot^2 \theta - 1}{2 \cot \theta}$
 50. $\cot(2\theta) = \frac{1}{2}(\cot \theta - \tan \theta)$ 51. $\sec(2\theta) = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$ 52. $\csc(2\theta) = \frac{1}{2} \sec \theta \csc \theta$
 53. $\cos^2(2u) - \sin^2(2u) = \cos(4u)$ 54. $(4 \sin u \cos u)(1 - 2 \sin^2 u) = \sin(4u)$ 55. $\frac{\cos(2\theta)}{1 + \sin(2\theta)} = \frac{\cot \theta - 1}{\cot \theta + 1}$
 56. $\sin^2 \theta \cos^2 \theta = \frac{1}{8}[1 - \cos(4\theta)]$ 57. $\sec^2 \frac{\theta}{2} = \frac{2}{1 + \cos \theta}$ 58. $\csc^2 \frac{\theta}{2} = \frac{2}{1 - \cos \theta}$
 59. $\cot^2 \frac{v}{2} = \frac{\sec v + 1}{\sec v - 1}$ 60. $\tan \frac{v}{2} = \csc v - \cot v$ 61. $\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$
 62. $1 - \frac{1}{2} \sin(2\theta) = \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta}$ 63. $\frac{\sin(3\theta)}{\sin \theta} - \frac{\cos(3\theta)}{\cos \theta} = 2$
 64. $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = 2 \tan(2\theta)$ 65. $\tan(3\theta) = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$
 66. $\tan \theta + \tan(\theta + 120^\circ) + \tan(\theta + 240^\circ) = 3 \tan(3\theta)$ 67. $\ln |\sin \theta| = \frac{1}{2}(\ln |1 - \cos(2\theta)| - \ln 2)$
 68. $\ln |\cos \theta| = \frac{1}{2}(\ln |1 + \cos(2\theta)| - \ln 2)$

In Problems 69–78, solve each equation on the interval $0 \leq \theta < 2\pi$.

69. $\cos(2\theta) + 6 \sin^2 \theta = 4$ 70. $\cos(2\theta) = 2 - 2 \sin^2 \theta$ 71. $\cos(2\theta) = \cos \theta$
 72. $\sin(2\theta) = \cos \theta$ 73. $\sin(2\theta) + \sin(4\theta) = 0$ 74. $\cos(2\theta) + \cos(4\theta) = 0$
 75. $3 - \sin \theta = \cos(2\theta)$ 76. $\cos(2\theta) + 5 \cos \theta + 3 = 0$
 77. $\tan(2\theta) + 2 \sin \theta = 0$ 78. $\tan(2\theta) + 2 \cos \theta = 0$

Mixed Practice

In Problems 79–90, find the exact value of each expression.

79. $\sin\left(2 \sin^{-1} \frac{1}{2}\right)$ 80. $\sin\left[2 \sin^{-1} \frac{\sqrt{3}}{2}\right]$ 81. $\cos\left(2 \sin^{-1} \frac{3}{5}\right)$ 82. $\cos\left(2 \cos^{-1} \frac{4}{5}\right)$
 83. $\tan\left[2 \cos^{-1}\left(-\frac{3}{5}\right)\right]$ 84. $\tan\left(2 \tan^{-1} \frac{3}{4}\right)$ 85. $\sin\left(2 \cos^{-1} \frac{4}{5}\right)$ 86. $\cos\left[2 \tan^{-1}\left(-\frac{4}{3}\right)\right]$
 87. $\sin^2\left(\frac{1}{2} \cos^{-1} \frac{3}{5}\right)$ 88. $\cos^2\left(\frac{1}{2} \sin^{-1} \frac{3}{5}\right)$ 89. $\sec\left(2 \tan^{-1} \frac{3}{4}\right)$ 90. $\csc\left[2 \sin^{-1}\left(-\frac{3}{5}\right)\right]$

In Problems 91–93, find the real zeros of each trigonometric function on the interval $0 \leq \theta < 2\pi$.

91. $f(x) = \sin(2x) - \sin x$ 92. $f(x) = \cos(2x) + \cos x$ 93. $f(x) = \cos(2x) + \sin^2 x$