this means that

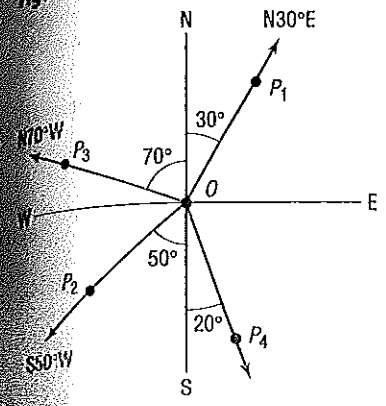
$$s = r\theta \approx (3960)(0.00588) \approx 23.3 \text{ miles}$$

In either case, it would seem that the brochure overstated the distance somewhat.

In navigation and surveying, the **direction** or **bearing** from a point O to a point P equals the acute angle θ between the ray OP and the vertical line through O, the north–south line.

Figure 16 illustrates some bearings. Notice that the bearing from O to P_1 is denoted by N30°E, indicating that the bearing is 30° east of north. In writing the bearing from O to P, the direction north or south always appears first, followed by an acute angle, followed by east or west. In Figure 16, the bearing from O to P_2 is S50°W, and the bearing from O to P_3 is N70°W.

Figure 16



EXAMPLE 11 **Finding the Bearing of an Object**

In Figure 16, what is the bearing from O to an object at P_4?

Solution The acute angle between the ray OP_4 and the north–south line through O is given as 20°. The bearing from O to P_4 is S20°E.

EXAMPLE 12 **Finding the Bearing of an Airplane**

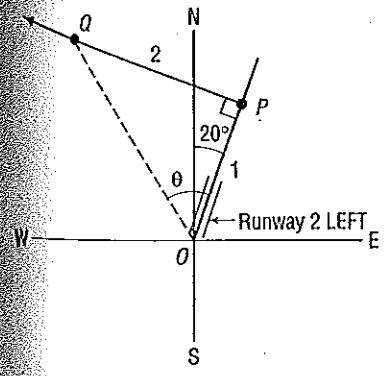
A Boeing 777 aircraft takes off from O'Hare Airport on runway 2 LEFT, which has a bearing of N20°E.* After flying for 1 mile, the pilot of the aircraft requests permission to turn 90° and head toward the northwest. The request is granted. After the plane goes 2 miles in this direction, what bearing should the control tower use to locate the aircraft?

Solution Figure 17 illustrates the situation. After flying 1 mile from the airport O (the control tower), the aircraft is at P. After turning 90° toward the northwest and flying 2 miles, the aircraft is at the point Q. In triangle OPQ, the angle θ obeys the equation

$$\tan\theta = \frac{2}{1} = 2 \quad \text{so} \quad \theta = \tan^{-1} 2 \approx 63.4°$$

The acute angle between north and the ray OQ is $63.4° - 20° = 43.4°$. The bearing of the aircraft from O to Q is N43.4°W.

Figure 17



Now Work PROBLEM 63

7.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. In a right triangle, if the length of the hypotenuse is 5 and the length of one of the other sides is 3, what is the length of the third side? (pp. A13–A15)
2. If θ is an acute angle, solve the equation $\tan\theta = \frac{1}{2}$. Express your answer in degrees, rounded to one decimal place. (pp. 482–485)
3. If θ is an acute angle, solve the equation $\sin\theta = \frac{1}{2}$. (pp. 482–485)

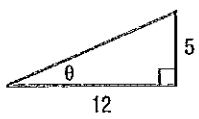
*In air navigation, the term **azimuth** denotes the positive angle measured clockwise from the north (N) to a ray OP. In Figure 16, the azimuth from O to P_1 is 30°; the azimuth from O to P_2 is 230°; and the azimuth from O to P_3 is 290°. In naming runways, the units digit is left off the azimuth. Runway 2 LEFT means the left runway with a direction of azimuth 20° (bearing N20°E). Runway 23 is the runway with azimuth 230° and bearing S50°W.

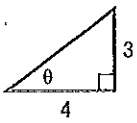
Concepts and Vocabulary

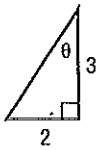
4. *True or False* $\sin 52° = \cos 48°$.
5. *True or False* In a right triangle, one of the angles is 90° and the sum of the other two angles is 90°.
6. When you look up at an object, the acute angle measured from the horizontal to a line-of-sight observation of the object is called the _____.
7. *True or False* In a right triangle, if two sides are known, then the triangle can be solved.
8. *True or False* In a right triangle, if the two acute angles are known, then the triangle can be solved.

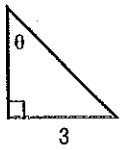
Skill Building

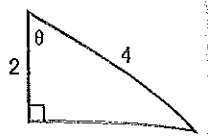
In Problems 9–18, find the exact value of the six trigonometric functions of the angle θ in each figure.

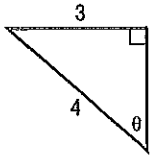
9. 

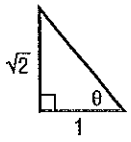
10. 

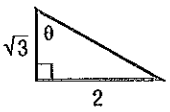
11. 

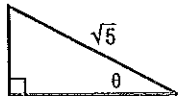
12. 

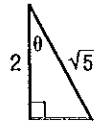
13. 

14. 

15. 

16. 

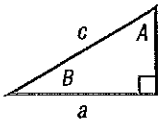
17. 

18. 

In Problems 19–28, find the exact value of each expression. Do not use a calculator.

19. $\sin 38° - \cos 52°$
20. $\tan 12° - \cot 78°$
21. $\dfrac{\cos 10°}{\sin 80°}$
22. $\dfrac{\cos 40°}{\sin 50°}$
23. $1 - \cos^2 20° - \cos^2 70°$
24. $1 + \tan^2 5° - \csc^2 85°$
25. $\tan 20° - \dfrac{\cos 70°}{\cos 20°}$
26. $\cot 40° - \dfrac{\sin 50°}{\sin 40°}$
27. $\cos 35° \sin 55° + \sin 35° \cos 55°$
28. $\sec 35° \csc 55° - \tan 35° \cot 55°$

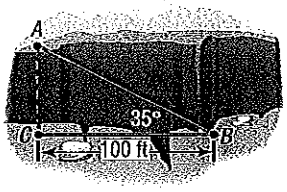
In Problems 29–42, use the right triangle shown below. Then, using the given information, solve the triangle.



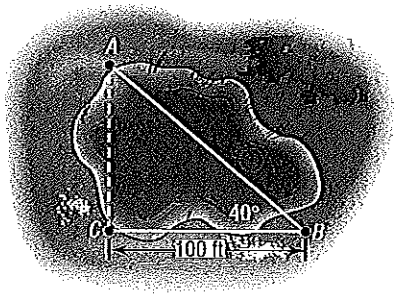
29. $b = 5$, $B = 20°$; find a, c, and A
30. $b = 4$, $B = 10°$; find a, c, and A
31. $a = 6$, $B = 40°$; find b, c, and A
32. $a = 7$, $B = 50°$; find b, c, and A
33. $b = 4$, $A = 10°$; find a, c, and B
34. $b = 6$, $A = 20°$; find a, c, and B
35. $a = 5$, $A = 25°$; find b, c, and B
36. $a = 6$, $A = 40°$; find b, c, and B
37. $c = 9$, $B = 20°$; find b, a, and A
38. $c = 10$, $A = 40°$; find b, a, and B
39. $a = 5$, $b = 3$; find c, A, and B
40. $a = 2$, $b = 8$; find c, A, and B
41. $a = 2$, $c = 5$; find b, A, and B
42. $b = 4$, $c = 6$; find a, A, and B

Applications and Extensions

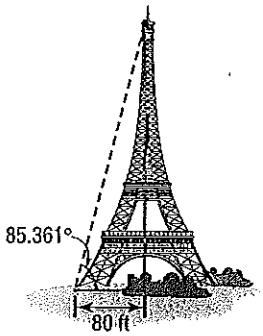
43. **Geometry** The hypotenuse of a right triangle is 5 inches. If one leg is 2 inches, find the degree measure of each angle.
44. **Geometry** The hypotenuse of a right triangle is 3 feet. If one leg is 1 foot, find the degree measure of each angle.
45. **Geometry** A right triangle has a hypotenuse of length 8 inches. If one angle is 35°, find the length of each leg.
46. **Geometry** A right triangle has a hypotenuse of length 10 centimeters. If one angle is 40°, find the length of each leg.
47. **Geometry** A right triangle contains a 25° angle.
    (a) If one leg is of length 5 inches, what is the length of the hypotenuse?
    (b) There are two answers. How is this possible?
48. **Geometry** A right triangle contains an angle of $\dfrac{\pi}{8}$ radian.
    (a) If one leg is of length 3 meters, what is the length of the hypotenuse?
    (b) There are two answers. How is this possible?
49. **Finding the Width of a Gorge** Find the distance from A to C across the gorge illustrated in the figure.



50. Finding the Distance across a Pond Find the distance from A to C across the pond illustrated in the figure.



51. The Eiffel Tower The tallest tower built before the era of television masts, the Eiffel Tower was completed on March 31, 1889. Find the height of the Eiffel Tower (before a television mast was added to the top) using the information given in the illustration.



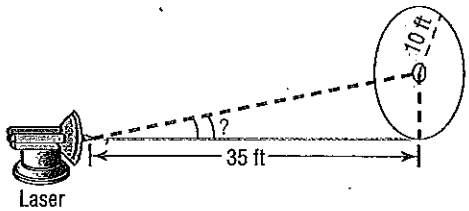
52. Finding the Distance of a Ship from Shore A person in a small boat, offshore from a vertical cliff known to be 100 feet in height, takes a sighting of the top of the cliff. If the angle of elevation is found to be 25°, how far offshore is the boat?

53. Finding the Distance to a Plateau Suppose that you are headed toward a plateau 50 meters high. If the angle of elevation to the top of the plateau is 20°, how far are you from the base of the plateau?

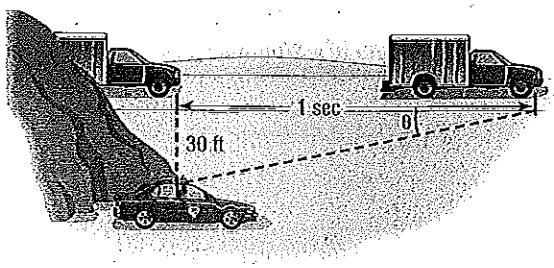
54. Finding the Reach of a Ladder A 22-foot extension ladder leaning against a building makes a 70° angle with the ground. How far up the building does the ladder touch?

55. Finding the Angle of Elevation of the Sun At 10 AM on May 21, 2013 a building 300 feet high cast a shadow 182 feet long. What was the angle of elevation of the Sun?

56. Directing a Laser Beam A laser beam is to be directed through a small hole in the center of a circle of radius 10 feet. The origin of the beam is 35 feet from the circle (see the figure). At what angle of elevation should the beam be aimed to ensure that it goes through the hole?



57. Finding the Speed of a Truck A state trooper is hidden 30 feet from a highway. One second after a truck passes, the angle θ between the highway and the line of observation from the patrol car to the truck is measured. See the illustration.



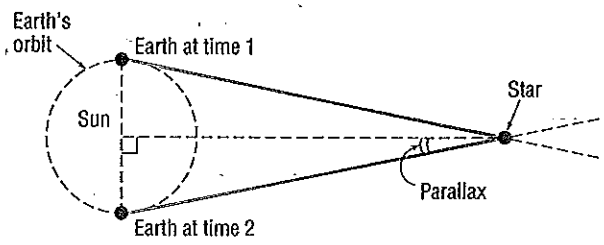
(a) If the angle measures 15°, how fast is the truck traveling? Express the answer in feet per second and in miles per hour.

(b) If the angle measures 20°, how fast is the truck traveling? Express the answer in feet per second and in miles per hour.

(c) If the speed limit is 55 miles per hour and a speeding ticket is issued for speeds of 5 miles per hour or more over the limit, for what angles should the trooper issue a ticket?

58. Security A security camera in a neighborhood bank is mounted on a wall 9 feet above the floor. What angle of depression should be used if the camera is to be directed to a spot 6 feet above the floor and 12 feet from the wall?

59. Parallax One method of measuring the distance from Earth to a star is the parallax method. The idea behind computing this distance is to measure the angle formed between Earth and the star at two different points in time. Typically, the measurements are taken so that the side opposite the angle is as large as possible. Therefore, the optimal approach is to measure the angle when Earth is on opposite sides of the Sun, as shown in the figure.



(a) Proxima Centauri is 4.22 light-years from Earth. If 1 light-year is about 5.9 trillion miles, how many miles is Proxima Centauri from Earth?

(b) The mean distance from Earth to the Sun is 93,000,000 miles. What is the parallax of Proxima Centauri?

60. Parallax See Problem 59. 61 Cygni, sometimes called Bessel's Star (after Friedrich Bessel, who measured the distance from Earth to the star in 1838), is a star in the constellation Cygnus.

(a) 61 Cygni is 11.14 light-years from Earth. If 1 light-year is about 5.9 trillion miles, how many miles is 61 Cygni from Earth?

(b) The mean distance from Earth to the Sun is 93,000,000 miles. What is the parallax of 61 Cygni?

61. Washington Monument The angle of elevation of the Sun is 35.1° at the instant the shadow cast by the Washington Monument is 789 feet long. Use this information to calculate the height of the monument.

62. Finding the Length of a Mountain Trail A straight trail with an inclination of 17° leads from a hotel at an elevation of 9000 feet to a mountain lake at an elevation of 11,200 feet. What is the length of the trail?

63. Finding the Bearing of an Aircraft A DC-9 aircraft leaves Midway Airport from runway 4 RIGHT, whose bearing is N40°E. After flying for $\frac{1}{2}$ mile, the pilot requests permission to turn 90° and head toward the southeast. The permission is granted. After the airplane goes 1 mile in this direction, what bearing should the control tower use to locate the aircraft?

64. Finding the Bearing of a Ship A ship leaves the port of Miami with a bearing of S80°E and a speed of 15 knots. After 1 hour, the ship turns 90° toward the south. After 2 hours, maintaining the same speed, what is the bearing to the ship from port?

65. Niagara Falls Incline Railway Situated between Portage Road and the Niagara Parkway directly across from the Canadian Horseshoe Falls, the Falls Incline Railway is a funicular that carries passengers up an embankment to Table Rock Observation Point. If the length of the track is 51.8 meters and the angle of inclination is 36°2′, determine the height of the embankment.

Source: www.niagaraparks.com

66. Willis Tower Willis Tower in Chicago is the tenth tallest building in the world and is topped by a high antenna. A surveyor on the ground makes the following measurement:

1. The angle of elevation from his position to the top of the building is 34°.
2. The distance from his position to the top of the building is 2593 feet.
3. The distance from his position to the top of the antenna is 2743 feet.

(a) How far away from the (base of the) building is the surveyor located?

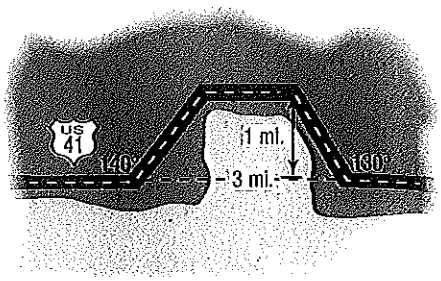
(b) How tall is the building?

(c) What is the angle of elevation from the surveyor to the top of the antenna?

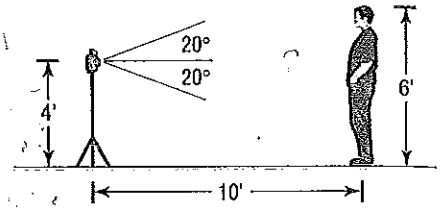
(d) How tall is the antenna?

Source: www.emporis.com

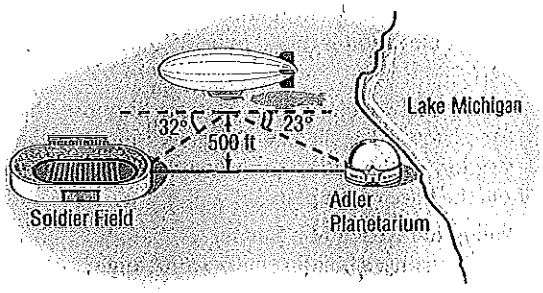
67. Constructing a Highway A highway whose primary directions are north–south is being constructed along the west coast of Florida. Near Naples, a bay obstructs the straight path of the road. Since the cost of a bridge is prohibitive, engineers decide to go around the bay. The illustration shows the path that they decide on and the measurements taken. What is the length of highway needed to go around the bay?



68. Photography A camera is mounted on a tripod 4 feet high at a distance of 10 feet from George, who is 6 feet tall. See the illustration. If the camera lens has angles of depression and elevation of 20°, will George's feet and head be seen by the lens? If not, how far back will the camera need to be moved to include George's feet and head?



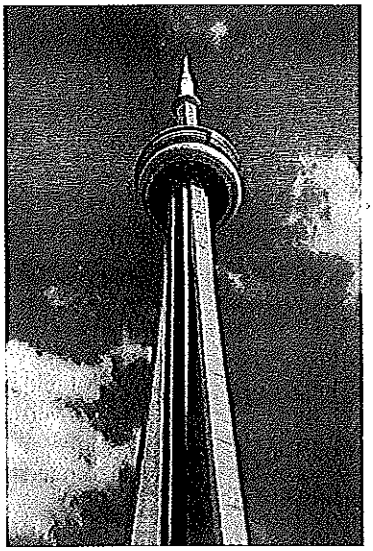
69. Finding the Distance between Two Objects A blimp, suspended in the air at a height of 500 feet, lies directly over a line from Soldier Field to the Adler Planetarium on Lake Michigan (see the figure). If the angle of depression from the blimp to the stadium is 32° and that from the blimp to the planetarium is 23°, find the distance between Soldier Field and the Adler Planetarium.



70. Hot-Air Balloon While taking a ride in a hot-air balloon in Napa Valley, Francisco wonders how high he is. To find out, he chooses a landmark that is to the east of the balloon and measures the angle of depression to be 54°. A few minutes later, after traveling 100 feet east, the angle of depression to the same landmark is determined to be 61°. Use this information to determine the height of the balloon.

71. Mt. Rushmore To measure the height of Lincoln's caricature on Mt. Rushmore, two sightings 800 feet from the base of the mountain are taken. If the angle of elevation to the bottom of Lincoln's face is 32° and the angle of elevation to the top is 35°, what is the height of Lincoln's face?

72. **The CN Tower** The CN Tower, located in Toronto, Canada, is the tallest structure in the Americas. While visiting Toronto, a tourist wondered what the height of the tower above the top of the Sky Pod is. While standing 4000 feet from the tower, she measured the angle to the top of the Sky Pod to be 20.1°. At this same distance, the angle of elevation to the top of the tower was found to be 24.4°. Use this information to determine the height of the tower above the Sky Pod.

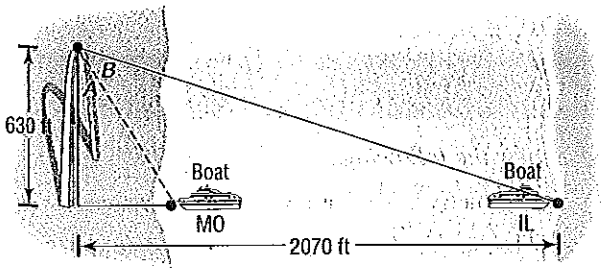


73. **Chicago Skyscrapers** The angle of inclination from the base of the John Hancock Center to the top of the main structure of the Willis Tower is approximately 10.3°. If the main structure of the Willis Tower is 1450 feet tall, how far apart are the two skyscrapers? Assume the bases of the two buildings are at the same elevation.

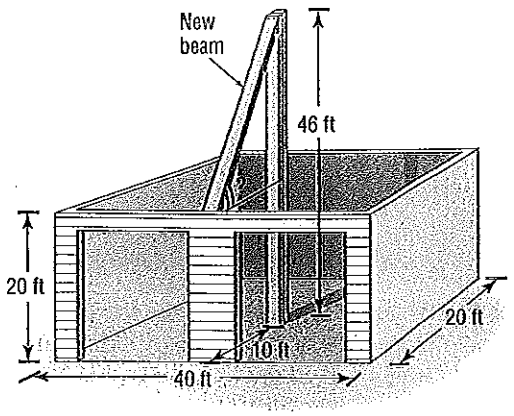
Source: www.emporis.com

74. **Estimating the Width of the Mississippi River** A tourist at the top of the Gateway Arch (height, 630 feet) in St. Louis, Missouri, observes a boat moored on the Illinois side of the Mississippi River 2070 feet directly across from the Arch. She also observes a boat moored on the Missouri side directly across from the first boat (see diagram). Given that $B = \cot^{-1}\frac{67}{55}$, estimate the width of the Mississippi River at the St. Louis riverfront.

Source: U.S. Army Corps of Engineers

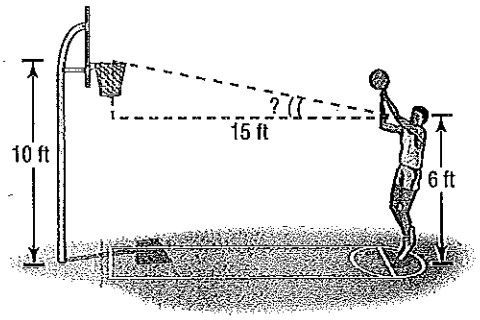


75. **Finding the Pitch of a Roof** A carpenter is preparing to put a roof on a garage that is 20 feet by 40 feet by 20 feet. A steel support beam 46 feet in length is positioned in the center of the garage. To support the roof, another beam will be attached to the top of the center beam (see the figure). At what angle of elevation is the new beam? In other words, what is the pitch of the roof?

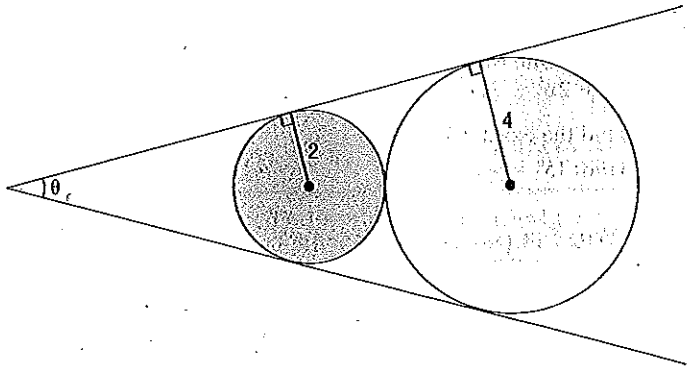


76. **Shooting Free Throws in Basketball** The eyes of a basketball player are 6 feet above the floor. The player is at the free-throw line, which is 15 feet from the center of the basket rim (see the figure). What is the angle of elevation from the player's eyes to the center of the rim?

[**Hint:** The rim is 10 feet above the floor.]



77. **Geometry** Find the value of the angle θ in degrees rounded to the nearest tenth of a degree.



78. **Surveillance Satellites** A surveillance satellite circles Earth at a height of h miles above the surface. Suppose that d is the distance, in miles, on the surface of Earth that can be observed from the satellite. See the illustration on the following page.

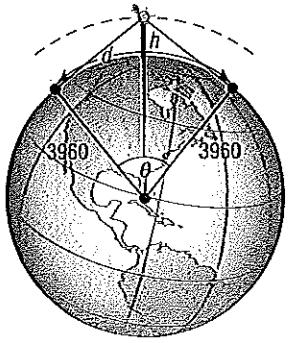
(a) Find an equation that relates the central angle θ to the height h.

(b) Find an equation that relates the observable distance d and θ.

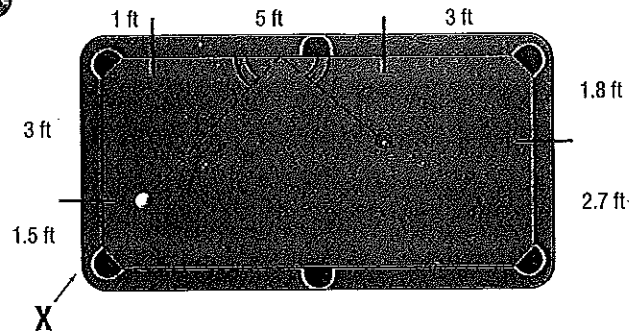
(c) Find an equation that relates d and h.

(d) If d is to be 2500 miles, how high must the satellite orbit above Earth?

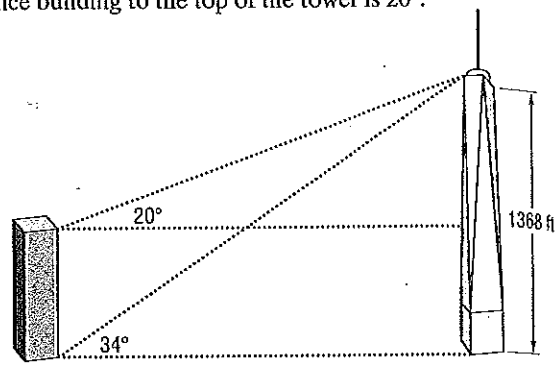
(e) If the satellite orbits at a height of 300 miles, what distance d on the surface can be observed?



79. Calculating Pool Shots A pool player located at **X** wants to shoot the white ball off the top cushion and hit the red ball dead center. He knows from physics that the white ball will come off a cushion at the same angle as it hits a cushion. Where on the top cushion should he hit the white ball?



80. One World Trade Center One World Trade Center (1WTC) is the centerpiece of the rebuilding of the World Trade Center in New York City. The tower is 1368 feet tall (not including a broadcast antenna). The angle of elevation from the base of an office building to the top of the tower is 34°. The angle of elevation from the helipad on the roof of the office building to the top of the tower is 20°.



(a) How far away is the office building from 1WTC? Assume the side of the tower is vertical. Round to the nearest foot.

(b) How tall is the office building? Round to the nearest foot.

Discussion and Writing

81. Explain how you would measure the width of the Grand Canyon from a point on its ridge.

82. Explain how you would measure the height of a TV tower that is on the roof of a tall building.

83. The Gibb's Hill Lighthouse, Southampton, Bermuda In operation since 1846, the Gibb's Hill Lighthouse stands 117 feet high on a hill 245 feet high, so its beam of light is 362 feet above sea level. A brochure states that ships 40 miles away can see the light and planes flying at 10,000 feet can see it 120 miles away. Verify the accuracy of these statements. What assumption did the brochure make about the height of the ship?

Retain Your Knowledge

Problems 84–87 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

84. Determine whether $x - 3$ is a factor of $x^4 + 2x^3 - 21x^2 + 19x - 3$.

85. Find the exact value of $\sin 15°$.
Hint: $15° = 45° - 30°$

86. Evaluate $\dfrac{f(x) - f(4)}{x - 4}$, where $f(x) = \sqrt{x}$ for $x = 5$, 4.5, and 4.1. Round results to three decimal places.

87. Solve $2\sin^2\theta - \sin\theta - 1 = 0$ for $0 \le \theta < 2\pi$.

'Are You Prepared?' Answers

1. 4 **2.** 26.6° **3.** 30°

7.2 The Law of Sines

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Trigonometric Equations (Section 6.3, pp. 482–485)
- Difference Formula for the Sine Function (Section 6.5, p. 502)
- Geometry Essentials (Appendix A, Section A.2, pp. A13–A19)

Now Work the **'Are You Prepared?'** problems on page 551.

OBJECTIVES
1 Solve SAA or ASA Triangles (p. 546)
2 Solve SSA Triangles (p. 546)
3 Solve Applied Problems (p. 549)