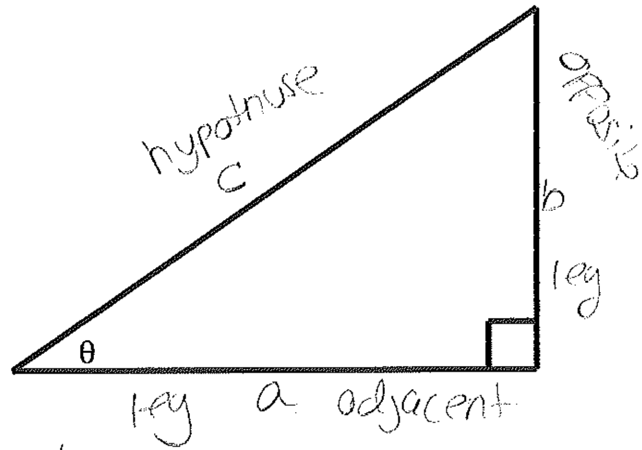
Pre Calc

7.1 Right Triangle Trigonometry

A triangle with one $\underline{90^\circ}$ angle is called a right triangle. A right triangle has two <u>legs</u>, which are the sides that make the right angle. The third side is called the <u>hypotenuse</u>, which is the side opposite to the right angle. The Pythagorean Theorem tells us that $\underline{a^2+b^2=c^2}$.



The diagram also shows us the angle $\underline{\theta}$, which is an <u>acute</u> angle (less than 90°). With respect to θ, we can label the legs as <u>opposite</u> and <u>adjacent</u>. The trigonometric functions of θ can be expressed as <u>ratios</u> of the sides of a triangle. The **six trigonometric** functions are....

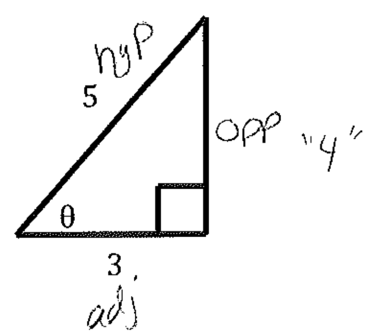
$$\sin\theta = \frac{Opposite}{Hypotnuse} = \frac{b}{c} \qquad \csc\theta = \frac{Hypotnuse}{Opposite} = \frac{c}{b} = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{Adjacent}{Hypotnuse} = \frac{a}{c} \qquad \sec\theta = \frac{Hypotnuse}{Adjacent} = \frac{c}{a} = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{Opposite}{Adjacent} = \frac{b}{a} \qquad \cot\theta = \frac{Adjacent}{Opposite} = \frac{a}{b} = \frac{1}{\tan\theta}$$

* *note that sin & cos are <u>cofunctions</u>, tan & cot are <u>cofunctions</u>, and sec & csc are <u>cofunctions</u>*

<u>Example</u>: Find the exact value of the six trigonometric functions of the angle θ in the figure below.



need length of "opp"

$(adj)^2 + (opp)^2 = (hyp)^2$

$(3)^2 + (opp)^2 = (5)^2$

$(opp)^2 = (5)^2 - (3)^2$

$(opp)^2 = 25 - 9$

$(opp)^2 = 16$

$opp = 4$

$\sin\theta = \frac{o}{h} = \frac{4}{5}$

$\cos\theta = \frac{a}{h} = \frac{3}{5}$

$\tan\theta = \frac{o}{a} = \frac{4}{3}$

$\csc\theta = \frac{h}{o} = \frac{5}{4}$

OR $= \frac{1}{\sin\theta} = \frac{1}{4/5} = \frac{5}{4}$

$\sec\theta = \frac{h}{a} = \frac{5}{3}$

OR $= \frac{1}{\cos\theta} = \frac{1}{3/5} = \frac{5}{3}$

$\cot\theta = \frac{a}{o} = \frac{3}{4}$

OR $= \frac{1}{\tan\theta} = \frac{1}{4/3} = \frac{3}{4}$

COMPLEMENTARY ANGLE THEOREM

Two acute angles are called complementary if their sum is 90°. The Complementary Angle Theorem states cofunctions of complementary angles are equal.

For example:

$\underbrace{\cos 30^\circ = \sin 60^\circ}_{\text{cofxns}}$ (complem. angles) $\quad \underbrace{\tan 40^\circ = \cot 50^\circ}_{\text{cofxns}}$ (complem angles) $\quad \underbrace{\sec 80^\circ = \csc 10^\circ}_{\text{cofxns}}$ (complem angles)

Example: Find the exact value of each expression without using a calculator.

a. $\underbrace{\tan 12^\circ - \cot 78^\circ}_{\text{cofxns}} = \tan 12^\circ - \tan 12^\circ$ b/c $\cot 78^\circ = \tan 12^\circ$

$= 0$

b. $\frac{\cos 40^\circ}{\sin 50^\circ} = \frac{\cos 40^\circ}{\cos 40^\circ} = 1$ b/c $\sin 50^\circ = \cos 40^\circ$

c. $\cos 20^\circ \sin 70^\circ + \sin 20^\circ \cos 70^\circ$ $\quad \cos 20^\circ = \sin 70^\circ$

$= \sin 70^\circ \sin 70^\circ + \cos 70^\circ \cos 70^\circ$ $\quad \sin 20^\circ = \cos 70^\circ$

$= \sin^2 70^\circ + \cos^2 70^\circ$

$= 1$

RECALL

Triangle Angle Sum Theorem: the sum of the interior angles of a triangle is 180°.

If $a^2 + b^2 = 1$, and a refers to the x value and b is the y value, then $x^2 + y^2 = 1$.

Further, if cosθ gives us the x value and sinθ gives us the y value, then $\cos^2\theta + \sin^2\theta = 1$.

notation $\begin{cases} (\cos\theta)^2 = \cos^2\theta \\ (\sin\theta)^2 = \sin^2\theta \end{cases}$

SOLVING A RIGHT TRIANGLE

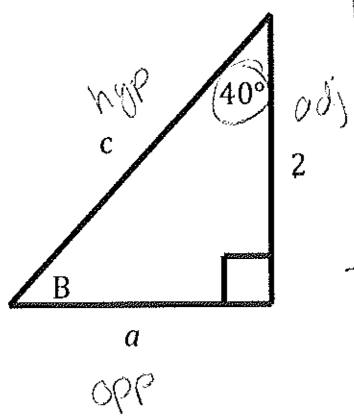
Solving a right triangle means to find the missing lengths of its sides and the measurements of its angles. For this section, make sure calculators is in *degree* mode. Solving a right triangle requires knowing one of the acute angles and one side OR two sides. Use a combination of Pythagorean Thrm, $S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$, and inverse trig fcns to solve.

$\theta = \sin^{-1}\left(\frac{o}{h}\right)$

$\theta = \cos^{-1}\left(\frac{a}{h}\right)$

$\theta = \tan^{-1}\left(\frac{o}{a}\right)$

Example. If $b = 2$ and $A = 40°$, find a, c, and B.



To find B:

$A + B = 90°$

$40° + B = 90°$

$-40° \quad -40$

$B = 50°$

To find a:

$\tan(40°) = \frac{a}{2}$

$a = 2\tan 40°$

$a \approx 1.68$

and

To find c:

$\cos(40°) = \frac{2}{c}$

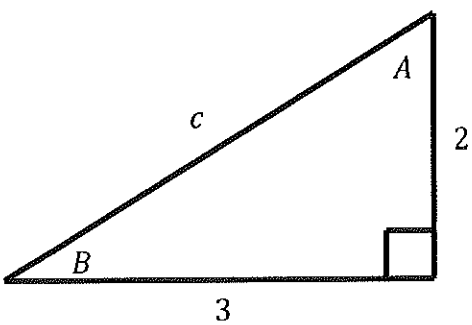
$\frac{\cos 40°}{1} = \frac{2}{c}$

$\frac{c \cdot \cos 40°}{\cos 40°} = \frac{2}{\cos 40}$

$\to c \approx 2.61$

Check w/ P.T.

Example. If $a = 3$ and $b = 2$, find c, A, and B.



Since we have 2 legs, use P.T to find c

$3^2 + 2^2 = c^2$

$9 + 4 = c^2$

$c = \sqrt{13} \approx 3.61$

To find A: $\tan A = \frac{3}{2}$

$A = \tan^{-1}(3/2)$

$= 56.31°$

To find B: $A + B = 90°$

$56.31° + B = 90°$

$B = 33.69°$

SOLVING APPLIED PROBLEMS

If a diagram is not provided, sketch one and label the given information before solving.

Example 1. A surveyor can measure the width of a river by setting up a transit at point C on one sie of the river and taking a sighting of a point A on the other side. Refer to the figure. After turning through an angle of 90° at C, the surveyor walks a distance of 200 meters to point B. Using the transit at B, the angle θ is measured and found to be 20°. What is the width of the river rounded to the nearest meter?

width of river is "b" ∴ solve for "b"

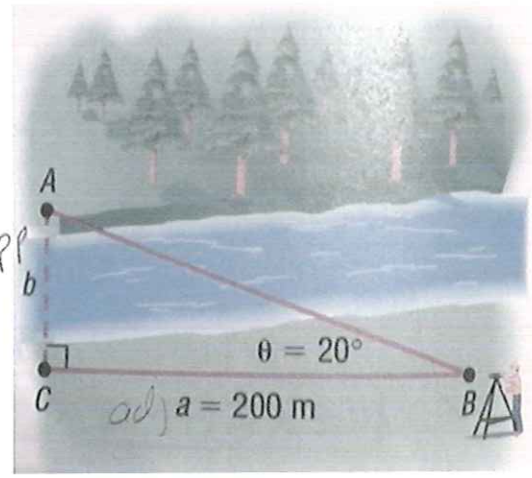
$\tan 20° = \frac{b}{200}$

$b = 200 \tan 20°$

≈ 72.79 meter

The width of the river is about 73 meters.

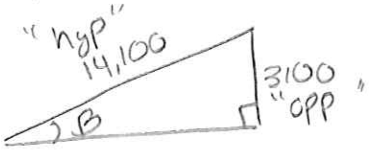
opp



Example 2. A straight trail leads from the Alpine Hotel, elevation 8000 feet, to a scenic overlook, elevation 11, 100 feet. The length of the tail is 14, 100 feet. What is the inclination (grade) of the trail? That is, what is the angle of B in the figure?

inclination refers to B

$y = 11{,}100 - 8000 = 3100$

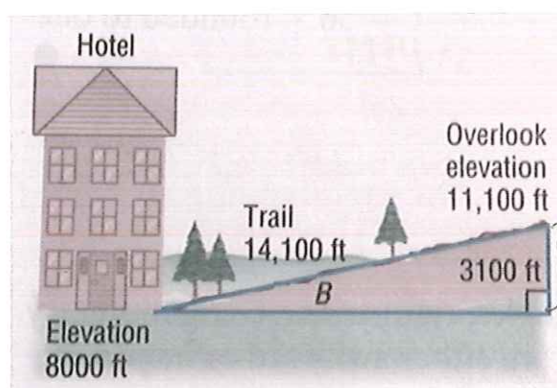


$\sin B = \frac{3100}{14{,}100}$

$B = \sin^{-1}\left(\frac{3100}{14{,}100}\right)$

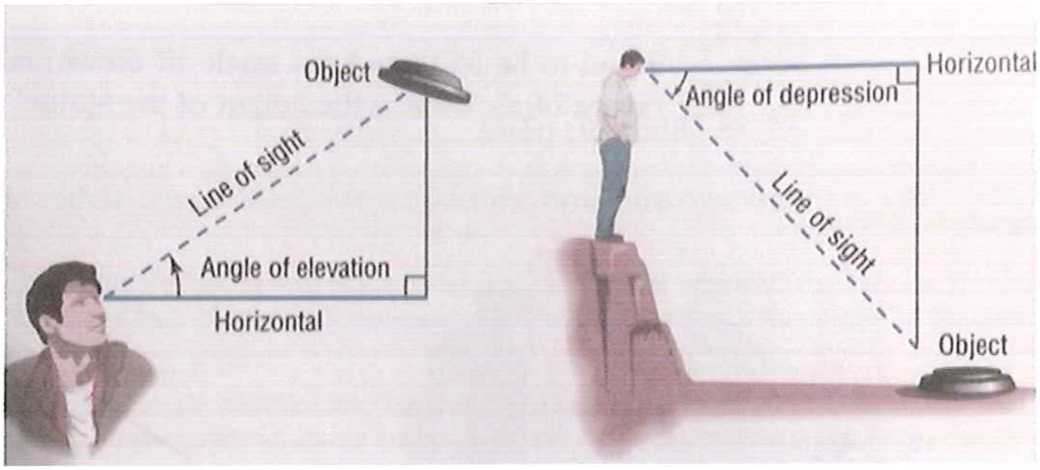
$\approx 12.7°$

The inclination of the trail is approximately 12.7°



y

ANGLE OF ELEVATION VS. ANGLE OF DEPRESSION



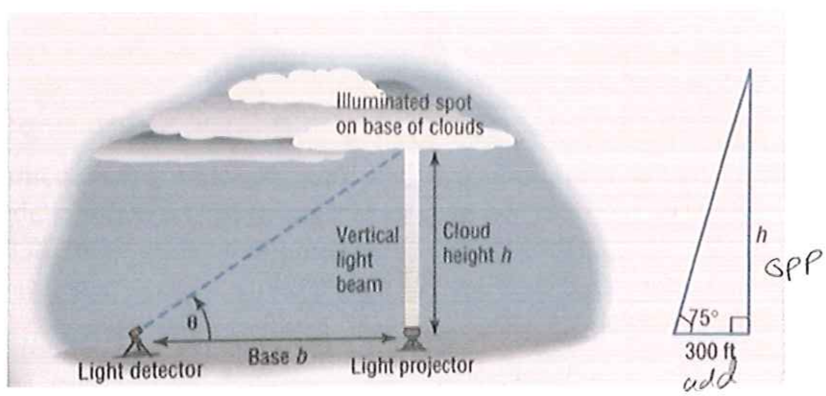
Example 3. Meteorologists find the height of a cloud using an instrument called a ceilometer. A ceilometer consists of a light projector that directs a vertical light beam up to the cloud base and a light detector that scans the cloud to detect the light beam. See the figure below. On May 3, 2013, at Midway Airport in Chicago, a ceilometer was employed to find the height of the cloud cover. It was set up with its light detector 300 feet from its light projector. If the angle of elevation from the light detector to the base of the cloud was 75°, what was the height of the cloud cover?

need to solve for "h"

$\tan 75^\circ = \frac{h}{300}$

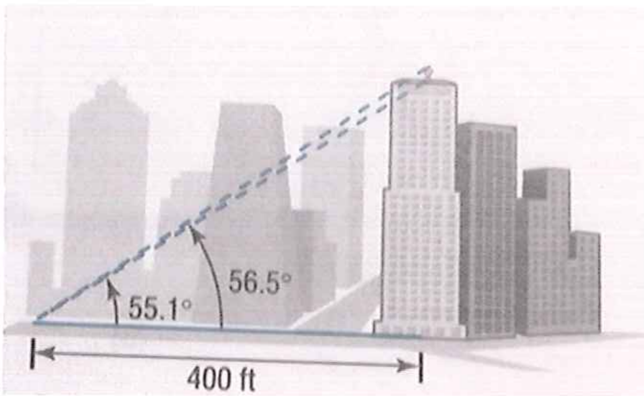
$h = 300 \tan 75^\circ$

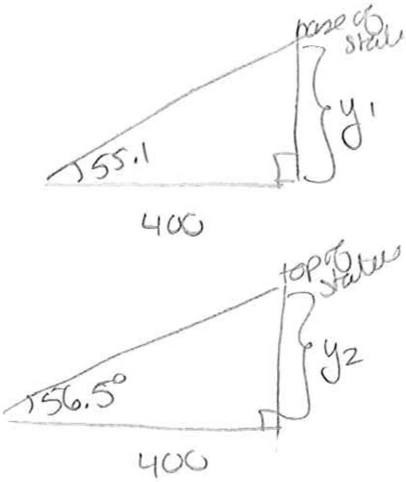
≈ 1120 ft



The height to the base of the cloud cover was approximately 1120 ft.

Example 4. Adorning the top of the Board of Trade building in Chicago is a statue of Ceres, the Roman goddess of wheat. From street level, two observations are taken 400 feet from the center of the building. The angle of elevation to the base of the statue is found to be 55.1° and the angle of elevation to the top of the statue is 56.5°. See the figure below. What is the height of the statue?





$y_2 - y_1 =$ height of statue

$\tan(55.1^\circ) = \frac{y_1}{400}$

$y_1 = 400\tan(55.1^\circ)$

≈ 573.39 ft

$\tan(56.5^\circ) = \frac{y_2}{400}$

$y_2 = 400\tan(56.5^\circ)$

≈ 604.33 ft

height of statue $= y_2 - y_1$

$= 604.33 - 573.39$

$= 30.94$

≈ 31 ft

The height of the statue is approximately 31 ft.