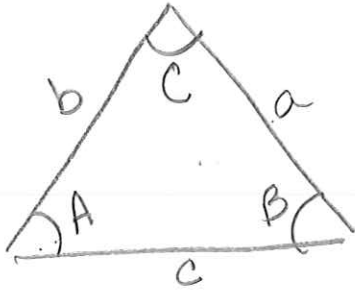


7.2 Law of Sine NOTES

Given a triangle ;



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

A, B, C are angles
a, b, c are sides

Law of Sine can be used to solve any Δ .

I. Given 2 angles + 1 side ; solve

find $\angle C$, a, b

$$\angle C = 180^\circ - (40^\circ + 45^\circ)$$

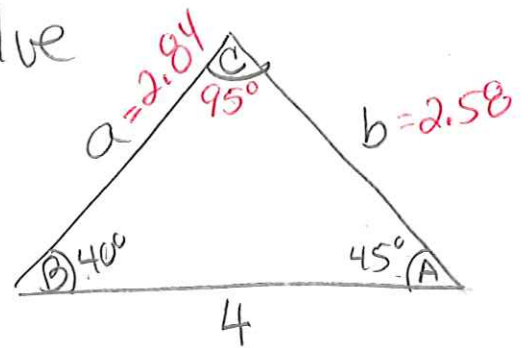
$$\angle C = 95^\circ$$

$$\frac{\sin 95^\circ}{4} = \frac{\sin 40^\circ}{b}$$

[use calc
in degree
mode]

$$\frac{b \sin 95^\circ}{\sin 95^\circ} = \frac{4 \sin 40^\circ}{\sin 95^\circ}$$

$$b \approx 2.58$$



$$\frac{\sin 45^\circ}{a} = \frac{\sin 95^\circ}{4}$$

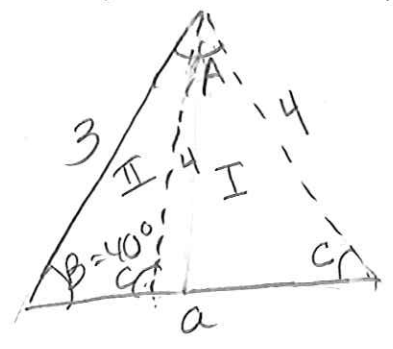
$$a = \frac{4 \sin 45^\circ}{\sin 95^\circ}$$

$$a \approx 2.84$$

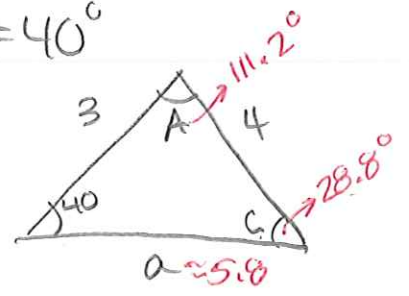
II Given 2 sides + 1 angle : Ambiguous Solns
 Involves using $\sin^{-1}(\theta) \Rightarrow$ 2 solns in QI + QII
 In turn, may have one Δ , two Δ s, or no Δ s.

ex) Determine whether the given info. results in 1, 2, or no Δ s. Solve each Δ that results.

Given: $b=4, c=3, B=40^\circ$



I



$$\frac{\sin C}{3} = \frac{\sin 40}{4}$$

$$4 \sin C = 3 \sin 40$$

$$\angle C = \sin^{-1} \left[\frac{3 \sin 40}{4} \right]$$

$$\angle C = 28.8^\circ$$

$$\angle A = 180^\circ - (40^\circ + 28.8^\circ) = 111.2^\circ$$

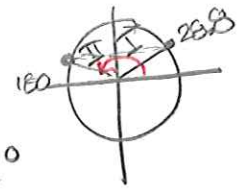
$$\frac{\sin(111.2^\circ)}{a} = \frac{\sin 40}{4}$$

$$a = \frac{4 \sin(111.2^\circ)}{\sin(40^\circ)}$$

$$a \approx 5.8$$

2nd in QII

$$180 - 28.8 = 151.2^\circ$$



NOT POSSIBLE

\therefore 1 triangle

ex) $b=2, c=3, B=100^\circ$

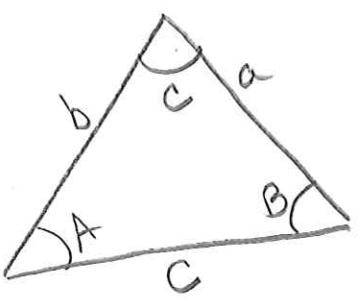
No Δ . $b < c$ $\angle B$ is largest angle, c is larger than $b \therefore$ not possible

7.3. Law of Cosine

Law of Sine is used to solve SAA or ASA triangles.

Law of Cosine is used to solve SAS or SSS triangles.

For a triangle with sides a, b, c and opposite angles A, B, C respectively

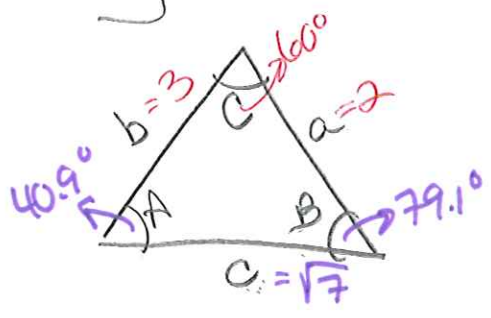
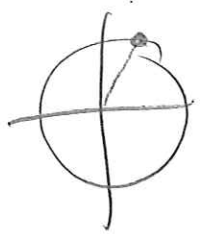


$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

ex) Solve the Δ : $a=2, b=3, C=60^\circ$



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= (2)^2 + (3)^2 - 2(2)(3) \cos 60^\circ$$

$$= 4 + 9 - 12(\frac{1}{2})$$

$$= 13 - 6$$

$$c^2 = 7$$

$$c = \sqrt{7}$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$9 = 4 + 7 - 2(2)(\sqrt{7}) \cdot \cos B$$

$$9 = 11 - 4\sqrt{7} \cos B$$

$$-2 = -4\sqrt{7} \cos B$$

$$\cos B = \frac{2}{4\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{14}$$

$B = 79.1^\circ$

$$\angle A = 180 - (79.1 + 60^\circ)$$

$$\angle A = 40.9^\circ$$

