8.2 Confidence Intervals for a Proportion

Learning Objectives:

1. State and check the Random, 10%, and Large Counts conditions for constructing a confidence interval for a population proportion.
2. Determine critical values for calculating a C % confidence interval for a population proportion using a table or technology.
3. Construct and interpret a confidence interval for a population proportion.
4. Determine the sample size required to obtain a C % confidence interval for a population proportion with a specified margin of error.

Vocabulary: critical value, standard error

ACTIVITY Hershey Kisses

Read 492–496

What are the three conditions for constructing a confidence interval for a proportion? What happens if one of the conditions is violated?

① Random: well-designed random sample or random assignments in experiments.

② Indipendence/10% condition: $n \leq \frac{1}{10}N$ / $10n \leq N$

③ Large Counts Condition: $n\hat{p} \geq 10$ $n(1-\hat{p}) \geq 10$ use $\hat{p}$ b/c we don't know p

* if any of the 3 conditions are violated, the actual capture % will be different than advertised.

Read 496–499

When the standard deviation of a statistic is estimated from data, the result is called the standard error of the statistic.

What is the difference between the standard deviation (σ) of a statistic and the standard error (SE) of a statistic?

Both measure typical distance from mean/center, but the σ is when we know p (population) & SE is when we know $\hat{p}$ (sample)

What is the formula for the standard error of the sample proportion? Is this formula on the formula sheet?

$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ Not on formula sheet — but similar to $\sigma_{\hat{p}}$ using $\hat{p}$ instead of p

What is a critical value? How is it calculated? What's up with the *?

critical value = z^* $\Rightarrow$ CI = Statistic $\pm (z^*)(SE)$

- measures how many SE we need to extend the interval to the desired confidence level
- "*" reminds us that it isn't a normal z-score calculation from data
- find z^* using z-table OR invNorm

Alternate Example: Find the critical value for a 96% confidence interval for a proportion.

using z-table: area closest to .02

$z^* = \pm 2.05$

OR

2nd VARS invNorm(area: .02, $\mu = 0$, $\sigma = 1$) or invNorm(.02) $= \pm 2.05$

What is the formula for a **one-sample *z* interval for a proportion**? Is this formula on the formula sheet?

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

critical value; $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \rightarrow SE$

point estimate; margin of error

NO! its just a more specific version of the CI formula

Alternate Example: Students in an AP Statistics class wants to estimate the proportion of pennies in circulation that are more than 10 years old. To do this, they gathered all the pennies they had in their pockets and purses. Overall, 57 of the 102 pennies they have are more than 10 years old.

(a) Identify the population and the parameter of interest.

Population: all pennies in circulation in U.S.

Parameter: proportion of all U.S. circulating pennies that are $\geq$ 10 yrs old.

(b) Check the conditions for calculating a confidence interval for the parameter.

✓ (1) Random - took a random sample of 102 pennies

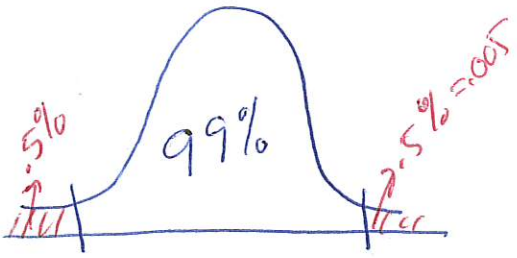
✓ (2) 10% - since $10(102) \leq$ Pop. of pennies in U.S. - we can assume this condition is met

✓ (3) large count - $n\hat{p} = (102)\left(\frac{57}{102}\right) = 57 \geq 10$

$n(1-\hat{p}) = (102)\left(1 - \frac{57}{102}\right) = 44.98 \geq 10$

$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $\quad \hat{p} = \frac{57}{102} \approx .5588$ $\quad n = 102$

(c) Construct a 99% confidence interval for the parameter.



Need z^*

using technology: invNorm(.005) = ±2.58

$SE = \sqrt{\frac{.5588(1-.5588)}{102}} \approx .049$

$.559 \pm 2.58(.049) = (.559 - .126, .559 + .126)$

$.126$

$(.433, .685)$

(d) Interpret the interval in context.

We are 99% confident that the true ~~popula~~ proportion of circulating pennies more than 10yrs old is between .433 and .685.

(e) Is it plausible that more than 60% of all pennies in circulation are more than 10 years old?

→ P

Yes, since .60 is in our interval, it is a plausible value of p.

th
end 3/22

HW page 490 (10, 20, 21–24), page 504 (27, 29, 31, 33, 34)

Read 499–501

What is the four-step process for calculating a confidence interval? What do you need to do in each step? Do you always have to do the four steps?

STATE: What parameter do you want to estimate and at what confidence level?

PLAN: Identify the appropriate inference method. Check conditions.

DO: If the conditions are met, perform calculations.

CONCLUDE: Interpret your interval in the context of the problem.

Is it OK to use your calculator to calculate the interval?

yes - but not recommended

- there are MC questions that look for use of formula
- No partial credit on FR if you don't show work
- Don't show two methods! Maybe use TI 84 to check work.

AP Exam Common Error: Many students use the 1-PropZInt feature in calculators to correctly calculate the confidence interval and then try to show their work with an incorrect formula. Using p rather than $\hat{p}$ is considered a wrong equation. Understand that using two methods (technology and formula) to calculate confidence intervals will result in having the worse response count.

Alternate Example: *Spinning the globe*

In her first-grade social studies class, Jordan learned that 70% of Earth's surface was covered in water. She wondered if this was really true and asked her dad for help. To investigate, he tossed an inflatable globe to her 50 times, being careful to spin the globe each time. When she caught it, he recorded where her right index finger was pointing. In 50 tosses, her finger was pointing to water 33 times. Construct and interpret a 95% confidence interval for the proportion of Earth's surface that is covered in water.

$\hat{p} = \frac{33}{50} = .66$

STATE We want to estimate p = true proportion of Earth's surface covered in water with 95% confidence.

PLAN We should use a one sample z interval for p if conditions are met.

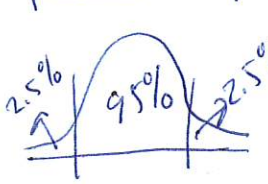
- Random - 50 locations that her finger was pointing to are random samples of all locations on the globe
- 10% - no need to check bc sampling was with replacement
- large counts: $n\hat{p} = 50(.66) = 33 \geq 10$ ✓
  $n(1-\hat{p}) = 50(1-.66) = 17 \geq 10$ ✓

DO $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \Rightarrow .66 \pm 1.96\left(\sqrt{\frac{.66(.34)}{50}}\right) = .66 \pm .131$

$= (.529, .791)$

to find z^*

invNorm(.025) = 1.96



CONCLUDE We are 95% confident that the interval (.529, .791) captures the true proportion of Earth's surface covered in water.

Read 501–503

What is the formula for the margin of error for a confidence interval for a proportion?

$$ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

How do you choose a value for $\hat{p}$ when solving for the sample size?

① use a guess for $\hat{p}$ based on a previous study

② use $\hat{p} = 0.5$ as the guess b/c the ME is largest @ $\hat{p} = 0.5$ ∴ this guess is conservative

To determine the sample size n that will yield C % confidence level interval for a population proportion p with a maximum margin of error ME, solve the following inequality for n:

Alternate Example: *Tattoos*

Suppose that you wanted to estimate p = the true proportion of students at your school who have a tattoo with 98% confidence and a margin of error of no more than 0.10. How many students should you survey?

Since we don't have any previous knowledge about the proportion of students w/ a tattoo, we will use $\hat{p} = 0.5$ to estimate the sample size

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le 0.10$$

divide 2.33

$$2.33 \sqrt{\frac{.5(1-.5)}{n}} \le .10$$

square both sides

$$\left(\sqrt{\frac{.25}{n}}\right)^2 \le \left(\frac{.10}{2.33}\right)^2$$

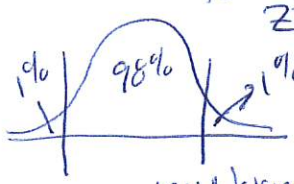
$$\frac{.25}{n} \le .001842$$

$$\frac{.25}{.001842} \le \frac{.001842n}{.001842}$$

$$135.7 \le n$$

$$n \ge 136$$

to find z^*:



invNorm(.01) ≈ 2.33

we need to survey at least 136 students to estimate the true proportion of students w/ a tattoo with 98% confidence and a ME at most 0.10.

HW page 505 (35–47 odd)