

## 8.3 Confidence Intervals for a Mean

### Learning Objectives:

1. State and check the Random, 10%, and Normal/Large Sample conditions for constructing a confidence interval for a population mean.
2. Explain how the  $t$  distributions are different from the standard Normal distribution and why it is necessary to use a  $t$  distribution when calculating a confidence interval for a population mean.
3. Determine critical values for calculating a  $C\%$  confidence interval for a population mean using a table or technology.
4. Construct and interpret a confidence interval for a population mean.
5. Determine the sample size required to obtain a  $C\%$  confidence interval for a population mean with a specified margin of error.

**Vocabulary:** confidence interval for the mean, standard error, degrees of freedom,  $t$  distribution, Normal/Large sample condition, one-sample  $t$  interval

### Activity

Rossman Chance Applet “*Simulating Confidence Intervals for a Population Parameter*”  
Choose “means” and “Normal” with  
 $\mu = 100, \sigma = 40, n = 4, interval = 100$  and *confidence level = 99%*.

Describe what happens after 1000 samples for each setting below. Why do some settings capture the true mean better than others?

- “z with  $\sigma$ ”
- “z with  $s$ ”
- “ $t$ ”

When should we use a  $t^*$  critical value rather than a  $z^*$  critical value for calculating a CI for a population mean?

*Read 510–514*

How do we calculate the value of  $t^*$  to use? How do we calculate degrees of freedom?

What is a  $t$  distribution, anyway? Describe the shape, center, and spread of the  $t$  distributions.

***Alternate Example:***  $t^*$  critical values

(a) Suppose you wanted to construct a 90% confidence interval for the mean  $\mu$  of a population based on an SRS of size 10. What critical value  $t^*$  should you use?

(b) What if you wanted to construct a 99% confidence interval for  $\mu$  using a sample of size 75?

*Read 514–517*

What are the three conditions for constructing a confidence interval for a population mean?

*Read 518–520*

What is the formula for the standard error of the sample mean? Is this formula on the formula sheet?

What is the formula for a confidence interval for a population mean? Is this formula on the formula sheet?

***Alternate Example: Milk's Favorite Cookie***

For their second semester project in AP<sup>®</sup> Statistics, Ann and Tori wanted to estimate the average weight of an Oreo cookie to determine if the average weight was less than advertised. They selected a random sample of 36 cookies and found the weight of each cookie (in grams). The mean weight was  $\bar{x} = 11.3921$  grams with a standard deviation of  $s_x = 0.0817$  grams.

(a) Construct and interpret a 95% confidence interval for the mean weight of an Oreo cookie.

(b) On the packaging, the stated serving size is 3 cookies (34 grams). Does the interval in part (a) provide convincing evidence that the average weight of an Oreo cookie is less than advertised? Explain.

### 8.3 Confidence Intervals for a Mean (cont'd)

Read 520–522

What should you do if you think the Normal/Large Sample condition isn't met?

How can you lose credit for the Normal/Large Sample condition on the AP Exam?

Can you use your calculator for the Do step? Are there any drawbacks to this?

***Alternate Example:*** *Can you spare a square?*

As part of their final project in AP Statistics, Christina and Rachel randomly selected 18 rolls of a generic brand of toilet paper to measure how well this brand could absorb water. To do this, they poured 1/4 cup of water onto a hard surface and counted how many squares it took to completely absorb the water. Here are the results from their 18 rolls:

29	20	25	29	21	24	27	25	24
29	24	27	28	21	25	26	22	23

Construct and interpret a 99% confidence interval for  $\mu$  = the mean number of squares of generic toilet paper needed to absorb 1/4 cup of water.

Read 522–524

How can we choose an appropriate sample size when we plan to calculate a confidence interval for a mean?

***Alternate Example:*** *How much homework?*

Administrators at your school want to estimate how much time students spend on homework, on average, during a typical week. They want to estimate  $\mu$  at the 90% confidence level with a margin of error of at most 30 minutes. A pilot study indicated that the standard deviation of time spent on homework per week is about 154 minutes. How many students need to be surveyed to estimate the mean number of minutes spent on homework per week with 90% confidence and a margin of error of at most 30 minutes?