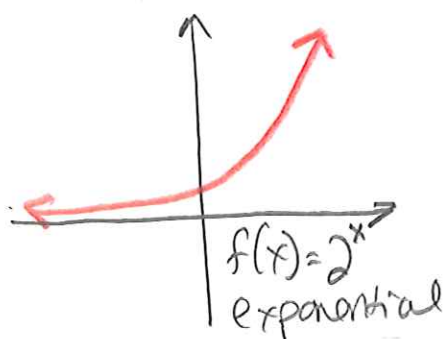
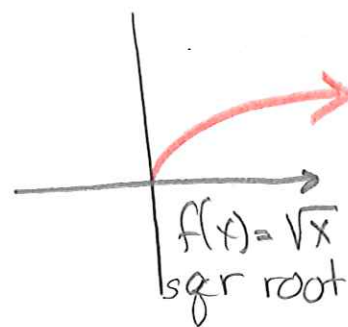
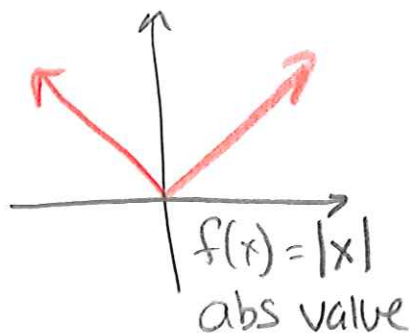
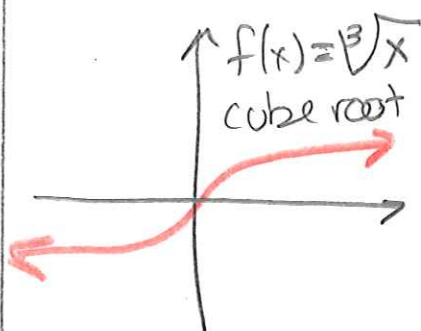
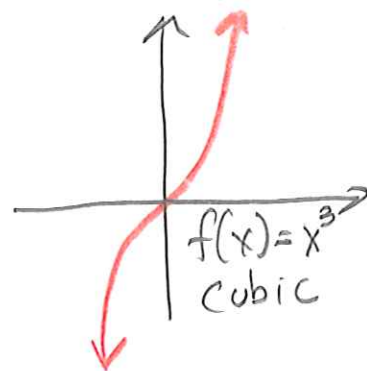
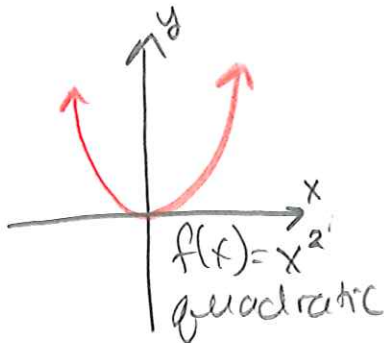
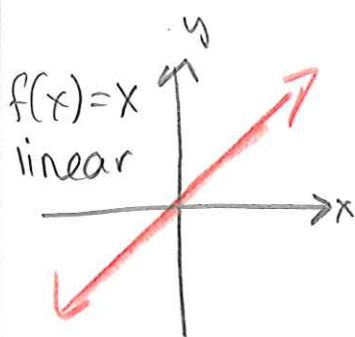


How do I transform functions?

Parent functions: linear, quadratic, cubic, cube root, abs value, square root



Each parent fxn can be transformed via shifts, reflections, and dilations. We consider vertical vs. horizontal transformations for all parent fxns except linear.

I. Horizontal Shifts

$$f(x+a)$$

h. shift left

ex) $\sqrt{x+2}$ $(x+2)^3$
 $(x+2)^2$ $\sqrt[3]{x+2}$
 $|x+2|$ 2^{x+2}

$$f(x-a)$$

h. shift right

ex) $\sqrt{x-2}$ $(x-2)^3$
 $(x-2)^2$ $\sqrt[3]{x-2}$
 $|x-2|$ 2^{x-2}

II Vertical Shifts

$$f(x)+a$$

vert. shift up

ex) $\sqrt{x}+2$ x^3+2
 x^2+2 $\sqrt[3]{x}+2$
 $|x|+2$ 2^x+2

$$f(x)-a$$

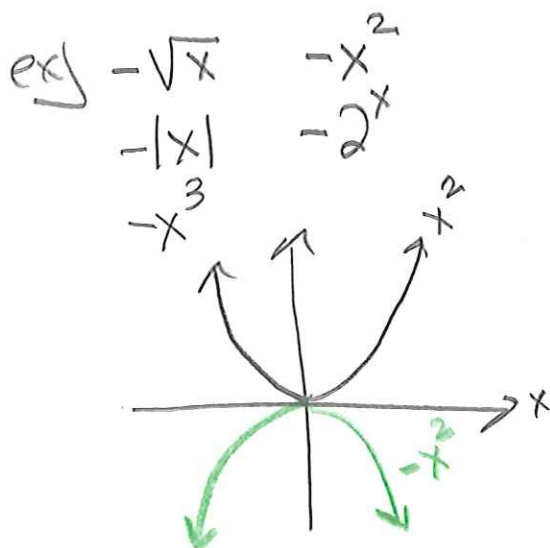
vert. shift down

ex) $\sqrt{x}-2$ x^3-2
 x^2-2 $\sqrt[3]{x}-2$
 $|x|-2$ 2^x-2

III Reflections

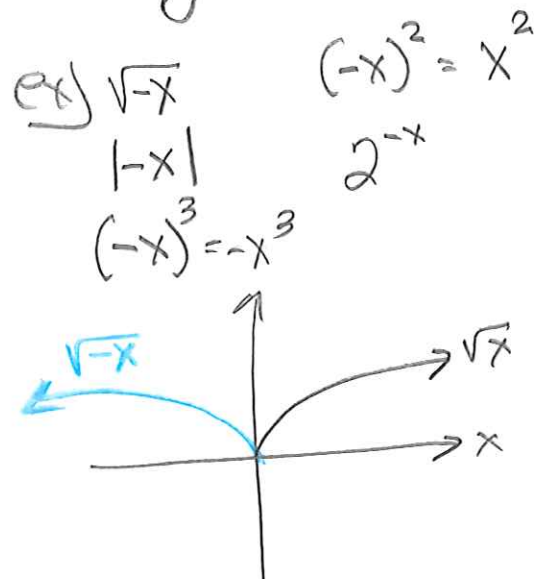
$$-f(x)$$

reflection about
x-axis



$$f(-x)$$

reflection about
y-axis



IV Dilations : "a" is the dilation factor

$a f(x)$: vertical

$a > 1$ ↙

Vertical stretch
"thinner" or "taller"

ex) $2x^2$ $5|x|$
 $3\sqrt{x}$ $2(2^x)$
 $4x^3$ $3\sqrt[3]{x}$

↘ $0 < a < 1$

V. Shrink or compression
"wider" or "shorter"

ex) $\frac{1}{2}x^2$
 $\frac{1}{3}\sqrt{x}$
 $\frac{1}{5}|x|$

$f(ax)$: horizontal

$a > 1$ ↙

horiz shrink or compression
"thinner" or "taller"

ex) $(3x)^2$
 $\sqrt{2x}$
 $|3x|$
 2^{3x}

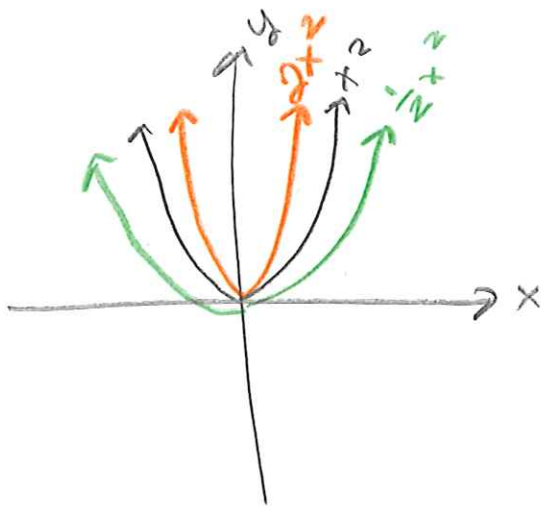
↘ $0 < a < 1$

horiz. stretch
"wider" or "shorter"

ex) $(\frac{1}{3}x)^2$
 $\sqrt{\frac{1}{2}x}$
 $|.15x|$
 $2^{.4x}$

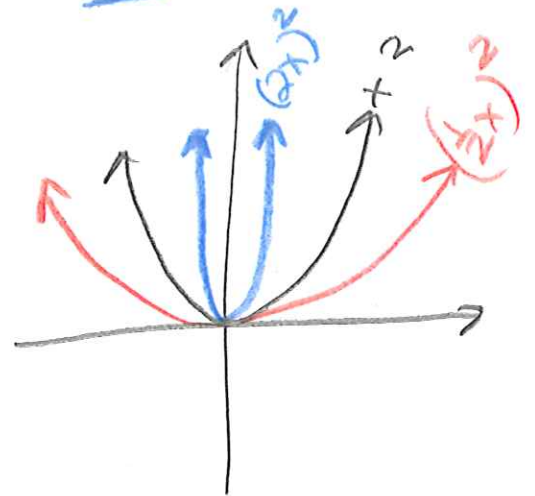
ex) v. dilations

$2x^2$ vs. $\frac{1}{2}x^2$



vs.

ex) h. dilation
 $(2x)^2$ vs. $(\frac{1}{2}x)^2$



In Summary: Let $g(x)$ be a transformation of $f(x)$

$$g(x) = \pm a f(\pm bx \pm c) \pm d$$

where $\pm \rightarrow$ reflection

$a \rightarrow$ vertical dilation

$b \rightarrow$ horizontal dilation

$c \rightarrow$ horizontal shift

$d \rightarrow$ vertical shift

Graphing: Start by graphing the parent fn.
Then graph one transformation at a time.

Next, graph the inner most transformation first and work outwards.

ex) $g(x) = -(x+2)^3 - 1$

① parent fn: $f(x) = x^3$

② horiz shift left by 2 units

③ reflection about x-axis

④ vertical shift down by 1 unit

