

# Advanced Algebra Learning Targets 2016-2017

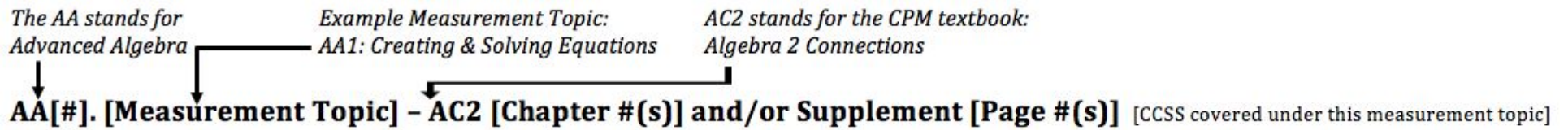
This document is the product of a team of PPS teachers experienced in writing learning targets and using them in instruction. While it represents our best work, we know this document will act as a working draft, to be revisited and revised as we continue to hone our instruction around CCSS Advanced Algebra.

The intended audience of this document is teachers of mathematics. While this document will be especially helpful for teachers who are using proficiency-based grading, it should also be useful to all teachers of CCSS Advanced Algebra as a summary of the new content students are expected to master due to Oregon's adoption of the Common Core State Standards for Mathematics. The learning targets are written in student-friendly language. We chose to further call out aspects of the learning target being assessed for teachers in the "procedural" and "conceptual" columns.

Every student should be expected to show mastery of ALL of the learning targets at a minimum level. A higher grade reflects a higher level of mastery. Our desire for this set of Learning Targets is that there is some consistency of expectations amongst and within buildings for students in PPS.

Proficiency based grading can be a complex and difficult process. If you plan to use these Measurement Topics and Learning Targets to track student progress, one way to make tracking more manageable is to test at the Measurement Topic Level, in which case students would need to pass all Learning Targets at a C level in order to pass the Measurement Topic. Individual Learning Targets could still be assessed formatively and re-testing.

We modeled our work after Robert J. Marzano's Measurement Topics (Formative Assessment & Standards-Based Grading, 2010). The structure is as follows:



Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA[#]a. Learning Target Text	This detail goes deeper into the more algorithmic type of problems students should be able to complete to demonstrate proficiency on the learning target.	This detail goes deeper into types of problem solving skills a student should be able to complete to demonstrate proficiency on this learning target.

<b>C</b> <i>Students can do</i>	<b>B</b> <i>Students can do</i>	<b>A</b> <i>Students can do</i>
These questions are examples of the minimum level of knowledge students need to demonstrate by the end of the course in order to earn a C for this Learning Target.	These questions are examples of more than the minimum level of knowledge students need to demonstrate by the end of the course in order to earn a B for this Learning Target. Students at this level can demonstrate a deeper level of understanding than the minimum expectation.	These questions are examples of a student who is exceeding mastery for this Learning Target. Often these questions require students to put multiple parts of learning together to solve a task or may reflect something that was never directly taught in the classroom.

+ Throughout this document this symbol (+) indicates an area that students do not need to master during this course. Teachers could use this as an extension or differentiation lesson.

## The Standards

The following *Common Core State Standards for Mathematics* are covered in the PPS CCSS Advanced Algebra course, including the recommended calendar and timeline (<https://sites.google.com/site/ppshighschoolmath/algebra>) and the Measurement Topics and Learning targets in this document. The standards covered are based on the recommendation in the CCSS *Mathematics Appendix A* Traditional Pathway. The complete set of standards and Appendix A are available for download at <http://corestandards.org/the-standards>.

The following are the **standards covered** in CCSS Advanced Algebra:

- The Mathematical Practices
- Number and Quantity
  - The Real Number System: N-CN.1, 2, 7
- Algebra
  - Seeing Structure in Expressions: A-SSE.1a, b, 2
  - Arithmetic with Polynomials and Rational Expressions: A-APR.1-4, 6
  - Creating Equations: A-CDE.1-4
  - Reasoning with Equations and Inequalities: A-REI.2
- Functions
  - Interpreting Functions: F-IF.4-5, 6b, e, 7b, c, e, 8b
  - Building Functions: F-BF.1b, 3, 4a
  - Linear, Quadratic, and Exponential Models: F-LE.4
  - Trigonometric Functions: F-TF. 1-2, 5, 8
- Statistics and Probability
  - Interpreting Categorical and Quantitative Data: S-ID.4
  - Making Inferences and Justifying Conclusions: S-IC. 1-6

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● AA1b. I can simplify and algebraically solve simple, rational and radical equations in one variable.	
<b>AA2. Graphs &amp; their Transformations</b>	<b>7-8</b>
.....	
● AA2a. I can recognize, describe, sketch and perform basic transformations.	
● AA2b. I can graph linear, quadratic, cubic, square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	
<b>AA3. Inverses</b>	<b>9</b>
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● AA3a. I can find the inverse of a function and represent and describe the relationship using tables, graphs, equations and domain and range.	
<b>AA4. Logarithms</b>	<b>10-11</b>
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● AA4a. I can use the definition of logarithms to evaluate logarithms and convert between logarithmic and exponential forms.	
● AA4b. I can interpret, model and graph exponential and logarithmic functions, showing intercepts and end behavior.	
<b>AA5. Trigonometric Functions</b>	<b>12-15</b>
.....	
● AA5a. I can extend the understanding of trigonometric functions using the unit circle in degrees & radians.	
● AA5b. I can interpret, model and graph periodic phenomena with trigonometric function.	
● AA5c. I can prove and use the Pythagorean Trig Identity: $\cos^2(x) + \sin^2(x) = 1$ .	

## AA6. Polynomials.....

### 16-19

- AA6a. I can perform arithmetic operations on polynomials.
- AA6b. I can understand the relationship between zeros and factors of polynomials.
- AA6c. I can prove polynomial identities.

## AA7. Complex

### Numbers.....20-22

- AA7a. I can perform arithmetic operations with complex numbers.
- AA7b. I can solve quadratic equations with real coefficients that have complex solutions.

## AA8. Statistics

### .....23-28

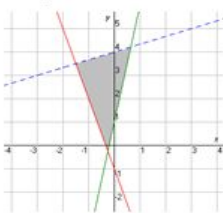
- AA8a. I can use the mean and the standard deviation of a data set to fit it to a normal distribution to estimate percentages and the area under the curve.
- AA8b. I can understand and evaluate random processes underlying statistical experiments.
- AA8c. I can make inferences and justify conclusions from sample surveys, experiments, and observational studies.

## AA1. Creating & Solving Equations ~ AC2 Ch. 2 and 5

A-CED. 1-4; A-APR. 6

\*\*Key learning targets for review (if needed): A1a. I can solve equations., A1b. I can manipulate equations., and A5b. I can solve quadratic equations.

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA1a. I can isolate a variable, manipulating equations with more than one variable.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Create equations and inequalities</li> <li><input type="checkbox"/> Solve for specified variable</li> <li><input type="checkbox"/> absolute value</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Determine the best method of simplifying the given rational expression</li> <li><input type="checkbox"/> Write equations/inequality to model the problem</li> </ul>

<p style="text-align: center;"><b>C</b> <i>Students can do</i></p>	<p>Solve for x:</p> $5x + 34 = -2(1 - 7x) \qquad x^2 + 5x + 6 = 0$ $3x + 5y = 21 \qquad 9x - 15 \leq 21$
<p style="text-align: center;"><b>B</b> <i>Students can do</i></p>	<p>Solve this system</p> $y < x^2 - 2x - 3$ $y \leq \frac{3}{4}x + 2$ <p>Solve for x:</p> $ 2x + 1  = 5$ <p style="text-align: right;">Write the system of inequalities for this shaded region</p> 
<p style="text-align: center;"><b>A</b> <i>Students can do</i></p>	<p>Solve for x: <math>2 3x - 1  + 5 = -2x + 8</math>      <math>y = c + \sqrt{x + r}</math></p>

### AA1. Creating & Solving Equations ~ AC2 Ch. 2 and 5

A-CED. 1-4

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
<p>AA1b. I can simplify and algebraically solve simple, rational and radical equations in one variable.</p>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Give examples showing how extraneous solutions may arise</li> <li><input type="checkbox"/> Rules of addition, subtraction, multiplication &amp; division</li> <li><input type="checkbox"/> Rewrite simple rational expressions</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Define extraneous solution &amp; generate examples of rational equations with extraneous solutions</li> <li><input type="checkbox"/> Determine which numbers cannot be solutions of a radical equation</li> </ul>

		and explain why they cannot be solutions <input type="checkbox"/> Generate examples of radical equations with extraneous solutions
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<b>C</b> <i>Students can do</i>	Solve for x: $\sqrt{(1 - 2x)} + 5 = 10$ $\sqrt{1-2x} = 10$ $\frac{x}{6} + \frac{2x}{3} = -2$
<b>B</b> <i>Students can do</i>	Solve for x: $x = \sqrt{56 - x}$ $\sqrt{37 - 3x} = x - 3$
<b>A</b> <i>Students can do</i>	Solve for x: $\frac{x+3}{x} - \frac{x-1}{x+3} = 23$ $\frac{4x-1}{x} = 3x$

## AA2. Graphs and their Transformations ~ AC2 Ch. 4

F.IF.4-6; F.BF.3

\*\*Key learning targets for review (if needed): A2a. and A5c. I can graph linear and quadratic functions. and A2b. I can determine the equation of a line. A4a., A3a., A5d., A7b. I can model linear functions, systems, quadratic functions and exponential functions in multiple ways.

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA2a. I can recognize, describe, sketch and perform basic transformations.	<input type="checkbox"/> Dilation <input type="checkbox"/> Reflection <input type="checkbox"/> Horizontal & vertical translations	<input type="checkbox"/> Recognize the key features of graphs and their equations. <input type="checkbox"/> Use and interpret function notation in its relation to transformations.

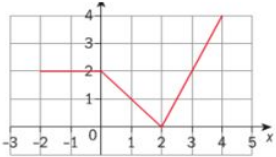
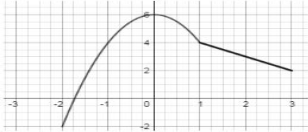
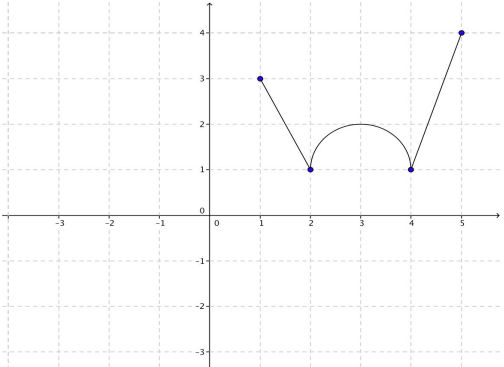
<p style="text-align: center;"><b>C</b> <i>Students can do</i></p>	<p>Accurately graph:</p> $y = \frac{-2}{x}$ $y = 3\sqrt{x-4}$
<p style="text-align: center;"><b>B</b> <i>Students can do</i></p>	<p>Show the differences between the graphs <math>y = x^3</math> and <math>y = -\frac{1}{4}(x + 2)^3 - 5</math> by sketching them on the same axes, and then by explaining how the numbers in the second equation alter the graph of the first equation.</p> <ol style="list-style-type: none"> <li>What is the maximum number of roots a polynomial of degree 3 can have? Sketch an example.</li> <li>What do you think is the maximum number of roots a polynomial of degree <math>n</math> can have?</li> <li>Can a polynomial of degree <math>n</math> have fewer than <math>n</math> roots? Under what conditions?</li> </ol>
<p style="text-align: center;"><b>A</b> <i>Students can do</i></p>	<p>Given <math>f(x)</math>. Sketch the graphs of <math>f(x)+3</math>, <math>4f(x)</math>, <math>f(x)-3</math></p>

## AA2. Graphs and their Transformations ~ AC2 Ch. 4

F.IF.4-6; F.BF.3

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA2b. I can graph linear, quadratic, cubic, square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Linear and quadratic</li> <li><input type="checkbox"/> Given the parent function, tell how it was transformed</li> <li><input type="checkbox"/> Using function notation</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Interpret functions in multiple representations that arise in real-world applications</li> <li><input type="checkbox"/> Interpret key features: intercepts, intervals where the function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</li> <li><input type="checkbox"/> Interpret the average rate of change</li> </ul>



<p style="text-align: center;"><b>C</b> <i>Students can do</i></p>	<p>Accurately draw by hand the graph: <math>y = \frac{1}{2}(x - 5)^2 + 6</math> including detail for vertex and two data points</p> <p>Write the equation(s) for the function:</p>	
<p style="text-align: center;"><b>B</b> <i>Students can do</i></p>	<p>Write a function that represents the graph.</p>	
<p style="text-align: center;"><b>A</b> <i>Students can do</i></p>	<p>Given <math>g(x)</math>, graph <math>g(-x)</math>, <math>-g(x)</math>, and <math>-g(-x)</math></p> 	<p>Graph this:</p> $f(x) = \begin{cases} -2x - 7 & \text{if } -5 \leq x \leq -3 \\ x + 2 & \text{if } -3 \leq x \leq -1 \\ x^2 & \text{if } -1 \leq x \leq 1 \\ -x + 2 & \text{if } 1 \leq x \leq 3 \\ 2x - 7 & \text{if } 3 \leq x \leq 5 \end{cases}$

### AA3. Inverses ~ AC2 Ch. 6

F-BF.4a & 4b (+)

\*\*Key learning targets for review (if needed): A9a. I can use function notation to evaluate and interpret functions. A9b. I can determine if a representation is a function and state its domain and range.

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA3a. I can find the inverse of a function and represent and describe the	<ul style="list-style-type: none"> <li><input type="checkbox"/> Write the inverse by solving <math>f(x) = c</math> for <math>x</math></li> <li><input type="checkbox"/> Write the inverse of a function in standard notation by replacing the <math>x</math></li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Define the inverse of a function</li> <li><input type="checkbox"/> (+) Use the composition of functions to verify that <math>g(x)</math> and <math>f(x)</math></li> </ul>



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<p style="text-align: center;"><b>C</b></p> <p><i>Students can do</i></p>	<p>Solve for x:</p> $5^x + 8 = 37$ $\log_9 x = 5$ <p>Compound/simple interest to find time of loan</p>
<p style="text-align: center;"><b>B</b></p> <p><i>Students can do</i></p>	<p>Solve for x:</p> $3 \log_5 x - 4 = 2$ <p>A problem with e or a natural log</p>
<p style="text-align: center;"><b>A</b></p> <p><i>Students can do</i></p>	<p><math>ab^{ct}=d</math> and <math>\log_b(d/a) = ct</math> are equivalent. Explain.</p>

## AA4. Logarithms ~ AC2 Ch. 6 and 7

F-LE.4; F.IF.7e, 8b; F-BF.1b

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA4b. I can interpret, model and graph exponential and logarithmic functions, showing intercepts and end behavior.**	<ul style="list-style-type: none"> <li>□ Identify percent rate of change</li> <li>□ Exponential growth or decay</li> </ul>	<ul style="list-style-type: none"> <li>□ Interpret functions in multiple representations that arise in real-world applications like combining functions</li> <li>□ Key features: intercepts, asymptotes, intervals where the function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; and end behavior</li> </ul>

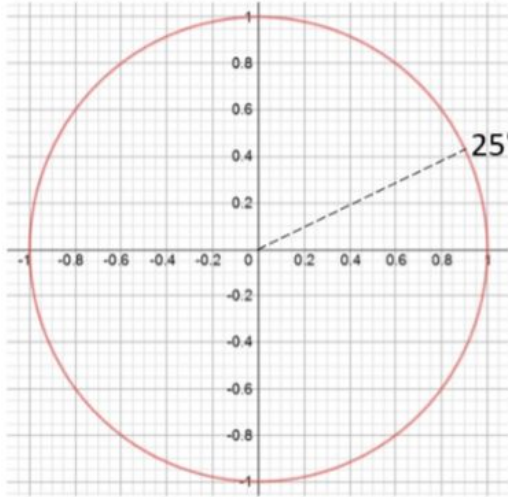
<b>C</b> <i>Students can do</i>	<p>Given the bank account function determine if it's growing or decaying. What is the growth/decay factor?</p> $M(t) = 583(0.94)^t$	<p>Graph:</p> $y = 2^{x+1} - 1$
<b>B</b> <i>Students can do</i>	<p>Graph <math>y = 2^x</math> and <math>y = \log_2 x</math>. Compare and contrast the two graphs. Then describe the graphs in at least three sentences. How are the graphs different and how are the graphs the same?</p>	
<b>A</b> <i>Students can do</i>	<p>Earthquake intensity is measured by the Richter scale. The formula for the Richter rating of a given quake is given by "<math>R = \log[I \div I_0]</math>" where <math>I_0</math> is the "threshold quake", or movement that can barely be detected, and the intensity <math>I</math> is given in terms of multiples of that threshold intensity.</p> <ul style="list-style-type: none"> <li>● You have a seismograph set up at home, and see that there was an event while you were out that had an intensity of <math>I = 989I_0</math>. A heavy truck rumbling by can cause a microquake with a Richter rating of 3 or 3.5. A "moderate" quake will have a Richter rating of 4 or more. Find the Richter rating. What was more likely to have happened, a microquake or a moderate quake?</li> </ul> <p>Scientific research has shown that the risk of having a car accident increases exponentially as the concentration of alcohol in the blood increases. A formula that models the risk of a car accident is <math>R = 6(2.72)^{12x}</math>, where <math>R</math> gives the percent of risk (as a whole number, not decimal) of having a car accident. The blood alcohol level is represented by <math>x</math>.</p> <p>a) What is the risk of having a car accident when the blood alcohol level, <math>x</math>, is zero?</p> <p>b) Find the blood alcohol level <math>x</math> that corresponds to a 25% risk. (solve algebraically and round your answer to 4 decimals)</p>	

\*\*Apply properties of logarithms to solve logarithmic and exponential equations or to simplify expressions has been moved to PreCalc – properties are not mentioned in CCSS

## AA5. Trigonometric Functions ~ AC2 Ch. 9 and AC2 Supplement

F-TF.1-2, 5, 8

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA5a. I can extend the understanding of trigonometric functions using the unit circle in degrees & radians.	<ul style="list-style-type: none"> <li>□ Define unit circle, central angle, co-terminal angle, intercepted arc, and +/- direction</li> <li>□ Relate a radian to the unit circle</li> <li>□ Use the unit circle on a coordinate plane to evaluate sine and cosine</li> </ul>	<ul style="list-style-type: none"> <li>□ Explain why co-terminal angles will all produce the same output when evaluated as the inputs of a trig function</li> <li>□ Use a similarity approach to find the radian measure of central angles in circles that are not unit circle</li> </ul>

<b>C</b> <i>Students can do</i>	<p>Given radian measure, find sin/cos value            Convert radians to degrees and back            Given ratio what is the angle (inverse)</p> <p>To the right is a circle with a radius of EXACTLY 1 inch. A central angle is marked.            Create a right triangle and answer the questions below. Opposite side:            Adjacent side:            Hypoteneuse: <math>\sin(\theta) = \cos(\theta) = \tan(\theta) =</math></p>	
<b>B</b> <i>Students can do</i>	<p>Given radian measure, find the tangent value</p> <p>What is the terminal angle for the central angle marked on the circle above. Show your work.</p> <p>The central angle of a circle is 135 degrees. What is the terminal angle? Show your work.</p>	
<b>A</b> <i>Students can do</i>	<p>Use negative angles outside of <math>2\pi</math></p> <p>For the central angle given on the circle above, what is the equivalent radian measure? Show your work.</p> <p>An angle around a circle is measure as 135 degrees. How many radians is that? Show your work.</p>	

AA5. Trigonometric Functions ~ AC2 Ch. 9 and AC2 Supplement

F-TF.1-2, 5, 7e. 8

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA5b. I can interpret, model and graph periodic phenomena with trigonometric function.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Use specified amplitude, frequency &amp; midline</li> <li><input type="checkbox"/> Explain the connection between frequency &amp; period</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Recognize &amp; write real world examples</li> <li><input type="checkbox"/> Examples: Speed of wheel, pendulums, springs</li> </ul>

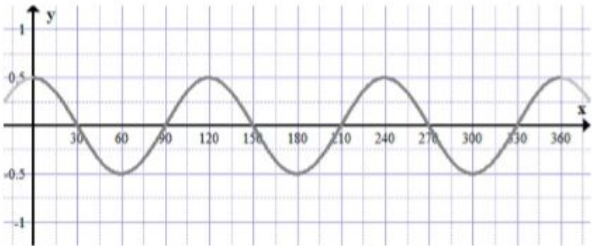
**C**

*Students can do*

Amplitude:

Period:

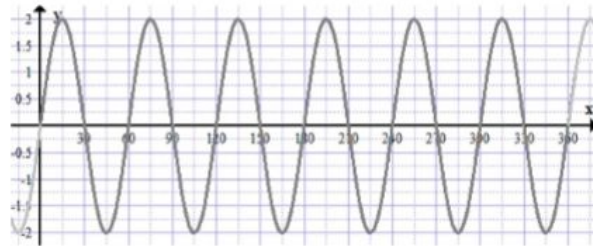
Frequency:



Amplitude:

Period:

Frequency:




Describe and sketch:  
 $y = -2 \sin(4\theta)$

Amplitude:

Period:

Frequency:



<p style="text-align: center;"><b>B</b></p> <p style="text-align: center;"><i>Students can do</i></p>	<ul style="list-style-type: none"> <li>● The height of a Ferris wheel at Oaks Amusement Park <math>y = 24 - 18\cos(180t)</math> where <math>t</math> is the time in minutes, and <math>y</math> is the height in feet.             <ol style="list-style-type: none"> <li>a. Give the height after 0.2 minute</li> <li>b. Find the period</li> <li>c. Find the height after 1.5 minutes</li> </ol> </li>   <li>● The cost per gallon of gas is modeled by <math>c = 1.25 - 0.05\cos(2\pi t)</math> cents/gallon where <math>t</math> is the time in days.             <ol style="list-style-type: none"> <li>a. What are the maximum and the minimum values for this function?</li> <li>b. How many days does it take for this function to repeat itself (What is the period)?</li> <li>c. What is the cost on day 12?</li> </ol> </li> </ul>
<p style="text-align: center;"><b>A</b></p> <p style="text-align: center;"><i>Students can do</i></p>	<p>Describe (in words) what this graph would look like. Include amplitude, frequency, period, and any shifts (translations).</p> <p><math>y = 15 \cos(2\theta) - 4</math></p> <p>Out of 360 days, the Earth travels around the sun in 365.25 days. The average radius of Earth's orbit around the sun is 92 million miles. Write a <math>\sin(\theta)</math> equation that models the Earth's orbit around the sun.</p> <p>The diameter of the moon's orbit is 1.5 million miles on average. After the full moon, it takes about 28 days for the moon to go through all its phases and become full again. Write a cosine equation that describes the motion of the moon.</p>

## AA5. Trigonometric Functions ~ AC2 Ch. 9 and AC2 Supplement

F-TF.1-2, 5, 8

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA5c. I can prove and use the Pythagorean Trig Identity: $\cos^2(x) + \sin^2(x) = 1$ .	<input type="checkbox"/> Use it to calculate	<input type="checkbox"/> Derive the Pythagorean Identity

<p><b>C</b> <i>Students can do</i></p>	<p>Use it to calculate Given <math>\sin x = \frac{3}{5}</math>, find <math>\cos x</math></p> <p>Use diagrams and words to show and explain how the Pythagorean Identity is derived from the unit circle.</p>
<p><b>B</b> <i>Students can do</i></p>	<p>Harder: when <math>x</math> is between <math>\pi</math> and <math>2\pi</math></p> <p>Given that <math>\sin x = \frac{\sqrt{3}}{7}</math> find <math>\cos x</math>, given that <math>x</math> is between <math>\pi</math> and <math>3\pi/2</math></p>
<p><b>A</b> <i>Students can do</i></p>	<p>Prove the following identity by making a clever substitution and using algebra:</p> $\frac{\cos^2(\theta) - 1}{\sin(\theta)} = -\sin(\theta)$



**AA6. Polynomials ~ AC2 Supplement**

A-APR.1, 6; A-SSE.1a, b. 2; F-IF.7c

Learning Target	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA6a. I can perform arithmetic operations on polynomials.	<ul style="list-style-type: none"> <li>□ Simplify higher degree polynomials using addition, subtraction, multiplication, and division</li> <li>□ Rewrite rational expressions through addition, subtraction, multiplication, and division.</li> <li>□ Use like denominators; combining like terms.</li> </ul>	<ul style="list-style-type: none"> <li>□ Apply the definition; Multiplying two polynomials always produces a polynomial</li> <li>□ Interpret key vocabulary like: terms, factor, coefficient</li> <li>□ Understand notation</li> </ul>

<p><b>C</b> Students can do</p>	<p>Use a horizontal format to add <math>2x^3 + x^2 - 5</math> and <math>x^2 + x + 6</math>.</p> <p>Use a vertical format to add <math>(5x^3 + 2x^2 - x + 7)</math>, <math>(3x^2 - 4x + 7)</math>, and <math>(-x^3 + 4x^2 - 8)</math>.</p> <p>51. <math>(10x^2 - 11) - (-7x^3 - 12x^2 - 15)</math></p> <p>52. <math>(15y^4 - 18y - 18) - (-11y^4 - 8y - 8)</math></p> <p>53. <math>5s - [6s - (30s + 8)]</math></p> <p>54. <math>3x^2 - 2[3x + (9 - x^2)]</math></p>	<p>a. <math>\frac{x^2}{x-5} - \frac{25}{x-5}</math></p> <p>b. <math>\frac{a^2}{a+5} + \frac{10a+25}{a+5}</math></p> <p>c. <math>\frac{x^2}{x-y} - \frac{2xy-y^2}{x-y}</math></p> <p><math>(4x^2 + x - 2)(5 + 3x - x^2)</math></p>
<p><b>B</b> Students can do</p>	<p>Simplify each of the expressions in parts (a) through (c) below. [ <b>a:</b> <math>b + a</math>, <b>b:</b> <math>3d + 2c^2</math>, <b>c:</b> <math>x - 1</math>, <b>d:</b> <math>xy</math> ]</p> <p>a. <math>ab(\frac{1}{a} + \frac{1}{b})</math>    b. <math>cd(\frac{3}{c} + \frac{2c}{d})</math>    c. <math>x(1 - \frac{1}{x})</math></p> <p>d. What expression would go in the box in order to make the equation <math>\square(\frac{5}{x} + \frac{8}{y}) = 5y + 8x</math> true?</p>	<p><math>\frac{3x^2 - 5x - 2}{2x^2 - 11x + 15} \cdot \frac{2x^2 - 5x}{3x^3 - 5x^2 - 2x}</math></p>
<p><b>A</b> Students can do</p>	<p>It is true that <math>\sqrt{(a-b)^2} \neq \sqrt{a^2 - b^2}</math>. Prove this statement in two different ways. First prove by substituting any numbers greater than 1 for <b>a</b> and <b>b</b>, and simplify. Then prove by factoring each expression under the radicals.</p> <p><b>For what value of <math>k</math> is <math>(x - 3)</math> a factor of the polynomial <math>p(x) = kx^3 - 4x^2 + 2x + 3</math>?</b></p>	

**AA6. Polynomials ~ AC2 Supplement**

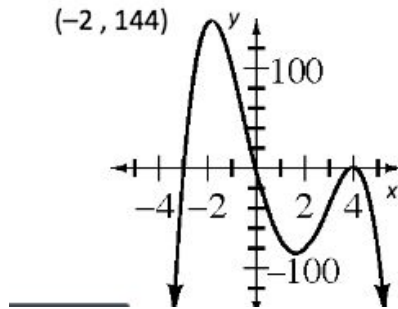
A-APR.2, 3; A-SSE.1a, b. 2; F-IF.7c

Learning Targets	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA6b. I can understand the relationship between zeros and factors of polynomials.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Divide using long division given the first root and using an area model to rewrite polynomials <b>with</b> and without remainders</li> <li><input type="checkbox"/> Find and use the zeros to sketch a rough graph of polynomials</li> <li><input type="checkbox"/> Factor polynomials completely</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Apply the Remainder Theorem to determine if a divisor <math>(x - a)</math> is a factor of the polynomial <math>p(x)</math></li> </ul>

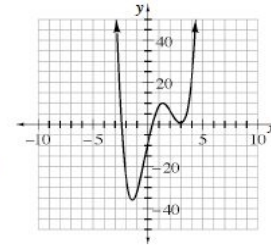
<p><b>C</b> <i>Students can do</i></p>	<p>Jessica is about to start the intermediate-level Polydoku puzzle shown at right. Show Jessica how to complete the puzzle. Make sure you can justify your solution. [ See completed puzzle below. ]</p> <div style="display: flex; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>A</td> <td>×</td> <td style="background-color: #cccccc;"></td> <td style="background-color: #cccccc;"></td> <td style="background-color: #cccccc;"></td> </tr> <tr> <td>B</td> <td>2x</td> <td></td> <td></td> <td></td> </tr> <tr> <td>C</td> <td>+ 5</td> <td></td> <td></td> <td></td> </tr> </table> <div style="margin-left: 10px;"> <math>6x^3 + 7x^2 - 16x + 10</math> </div> </div> <p>Use your results to complete the statements below.  <math>\left[ \frac{6x^3 + 7x^2 - 16x + 10}{2x + 5} = 3x^2 - 4x + 2 \text{ and } (2x + 5)(3x^2 - 4x + 2) = 6x^3 + 7x^2 - 16x + 10 \right]</math></p> <p><math>\frac{6x^3 + 7x^2 - 16x + 10}{2x + 5} = \underline{\hspace{2cm}}</math> and <math>(2x + 5) \cdot \underline{\hspace{2cm}} = \underline{\hspace{2cm}}</math></p> <p>Sketch a graph by hand: <math>P(x) = -(x + 5)(x + 1)^2(x - 3)</math></p> <p>Divide: <math>(2x^3 + x^2 - 19x + 36) \div (x + 4)</math></p>		1	2	3	4	A	×				B	2x				C	+ 5				<p>Consider the equation <math>5x^2 - 7x - 6 = 0</math> as you answer the questions in parts (a) through (d) below. [ a: <math>(x - 2)(5x + 3)</math>; b: <math>-\frac{3}{5}, 2</math>; c: Explanations vary; d: 3 and 2 are factors of 6, while 5 is a factor of the lead coefficient. ]</p> <ol style="list-style-type: none"> <li>a. What are the factors of <math>5x^2 - 7x - 6</math>?</li> <li>b. What are the solutions to the equation?</li> </ol> <p style="text-align: right;">Does <math>\frac{2x^3 + 15x^2 + 27x + 5}{2x + 5} = (x^2 + 5x + 1)</math>? Justify your answer.</p>
	1	2	3	4																		
A	×																					
B	2x																					
C	+ 5																					

**B**  
Students can do

write an equation for this polynomial:



Carlo was trying to factor the polynomial  $p(x) = x^4 - 4x^3 - 4x^2 + 24x - 9$  and find all of its roots. He had already found one factor by making a guess and trying it, so he had  $p(x) = (x - 3)(x^3 - x^2 - 7x + 3)$ . He was trying to factor  $x^3 - x^2 - 7x + 3$ , and he had tried  $(x + 3)$ ,  $(x + 1)$ , and  $(x - 1)$ , but none worked. Then Teo came by and said, "You should look at the graph." [ a: It shows that  $(x - 3)$  is a double factor and 3 is a double root. b:  $p(x) = (x - 3)^2(x^2 + 2x - 1)$ ,  $3, -1 \pm \sqrt{2}$  ]

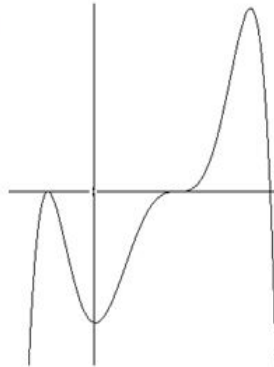


- How does the graph help?
- Complete the problem.

**A**  
Students can do

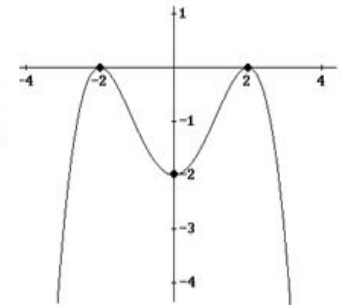
7. Give three different reasons why the graph at right cannot be the graph of  $p(x)$ , given that  $p(x) = x^4 - x^2 + 1$ .

- 
- 
- 



5. The graph of the polynomial  $y = ax^4 + bx^3 + cx^2 + dx + p$  is shown at right.

- Use your knowledge of roots and polynomials to find the values of coefficients  $a, b, c, d,$  and  $p$ . (the values could be positive, negative or zero)



Use the remainder theorem to find  $P(c)$  for the given  $P(x)$  and number  $c$ .

- $P(x) = 2x^3 - 7x^2 + 6x - 3, c = 3$
- $P(x) = x^3 + 6x^2 + 5x - 6, c = -2$



## AA7. Complex Numbers ~ AC2 Supplement

N-CN. 1, 2

Learning Targets	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA7a. I can perform arithmetic operations with complex numbers.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Define <math>i</math></li> <li><input type="checkbox"/> Able to perform <math>+/-</math> and multiplication of complex numbers</li> <li><input type="checkbox"/> Recognize patterns of powers of <math>i</math></li> <li><input type="checkbox"/> Identify that a complex number is written in the form <math>a + bi</math> where <math>a</math> and <math>b</math> are real numbers</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Justify properties using <math>i^2 = -1</math></li> </ul>

<b>C</b> <i>Students can do</i>	<p>Simplify each expression</p> <p>a. <math>(3+4i)+(7-2i)</math>                      b. <math>(3+5i)^2</math></p> <p>c. <math>(7+i)(7-i)</math>                              d. <math>(3i)(2i)^2</math></p>
<b>B</b> <i>Students can do</i>	<p>1) <math>i^{41}</math></p> <p>2) <math>\frac{(-3i^7)^2}{15i^8}</math></p> <p>3) <math>(-2i^3)^3(2i^5)</math></p>

## A

Students can do

So far you have been able to extend the rules for real numbers to add, subtract, and multiply complex numbers, but what about dividing? Can you use what you know about real numbers to divide one complex number by another? In other words, if a problem looks like this:

$$\frac{3+2i}{-4+7i}$$

what needs to be done to get an answer in the form of a single complex number,  $a+bi$ ?

Natalio had an idea. He said, "I'll bet we can use the conjugate!"

"How?" asked Ricki.

"Well, it's a fraction. Can't we multiply the numerator and denominator by the same number?" Natalio replied. [ a:  $\frac{3+2i}{-4+7i} \cdot \frac{-4-7i}{-4-7i} = \frac{2-29i}{65}$ , b:  $\frac{2}{65} - \frac{29}{65}i$  ]

- Natalio was not very clear in his explanation. Show Ricki what he meant they should do.
- Using Natalio's ideas you probably still came up with a fraction in part (a), but the denominator should be a whole number. To write a complex number such as  $\frac{c+di}{m}$  in the form  $a+bi$ , just use the distributive property to rewrite the result as  $\frac{c}{m} + \frac{d}{m}i$ . Rewrite your result for part (a) in this form.



**AA8. Statistics ~ AC2 Supplement**

S-ID.4

\*\* Key concepts to review (if needed): Five Number Summary.

Learning Targets	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA8a. I can use the mean and the standard deviation of a data set to fit it to a normal distribution to estimate percentages and the area under the curve.	<ul style="list-style-type: none"> <li><input type="checkbox"/> The 68-95-99.7 rule</li> <li><input type="checkbox"/> One, two, and three standard deviations of the mean</li> <li><input type="checkbox"/> Using calculator, table or spreadsheet</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Estimate the area under a normal curve using a calculator, table or spreadsheet</li> <li><input type="checkbox"/> Be able to answer or problems involving the question: When is normal distribution not applicable?</li> </ul>

**C**  
*Students can do*

The College Board scales the scores with a maximum of 800 and a minimum of 200.

- What is the corresponding range for the z-scores on the SAT's? (Assume a mean of 500 and standard deviation of 100.)
- Simone just got back her SAT results and received 800 on the verbal section. Does this imply that she did not skip any questions and answered all of the questions correctly? Explain.

Using the 68-95-99.7 Rule, determine the area of the shaded region under each of the curves below.

a)

b)

\*\*Teacher may shade in whatever region they see fit for problem above.

North City High School has served the following number of lunches since the beginning of the school year.

Number of lunches sold per day							
584	695	618	675	632	678	595	689
738	677	630	755	745	660	613	594
774	640	671	698	576	785	723	
721	603	636	663	665	640	671	
652	666	703	606	661	605	774	
							<i>checksum 24711</i>

- What are the mean and standard deviation? What is the five-number summary of the distribution?
- On your calculator, make a relative frequency histogram of the number of lunches served. Use an interval from 560 to 800 lunches, with a bin width of 40 lunches. Sketch the histogram and label the height of each of the bins.
- Describe the distribution. Make sure you consider the center, shape, spread, and outliers.



<p style="text-align: center;"><b>B</b> <i>Students can do</i></p>	<p>The 40 women in the sample recorded their race times in various charity 5K races in the table below.</p> <table border="1" data-bbox="464 180 835 310"> <thead> <tr> <th colspan="8">race time (min)</th> </tr> </thead> <tbody> <tr> <td>22.2</td><td>22.6</td><td>24.9</td><td>23.5</td><td>22.8</td><td>23.3</td><td>23.1</td><td>21.6</td> </tr> <tr> <td>21.3</td><td>22.9</td><td>25.7</td><td>23.3</td><td>23.3</td><td>22.5</td><td>24.4</td><td>22.7</td> </tr> <tr> <td>24.1</td><td>23</td><td>22.5</td><td>23.2</td><td>24.7</td><td>24.4</td><td>23.3</td><td>23.5</td> </tr> <tr> <td>23.1</td><td>22.5</td><td>22.3</td><td>22.6</td><td>23.6</td><td>23.3</td><td>23.3</td><td>23.4</td> </tr> <tr> <td>23.0</td><td>23.1</td><td>24.5</td><td>23.9</td><td>20.6</td><td>23.5</td><td>22.8</td><td>24.4</td> </tr> </tbody> </table> <p style="text-align: center;"><i>checksum 928.7</i></p> <ol style="list-style-type: none"> <li>Find the mean and standard deviation of the races times to four decimal places. Justify your choice of standard deviation.</li> <li>Create a relative frequency histogram with your calculator, and sketch it. Use an interval from 19 to 27 with a bin width of 1. Label the top of each bar with its relative frequency.</li> <li>Use the relative frequencies on your histogram to calculate the percentage of racers that had a time faster than 22 minutes. Remember, smaller times are faster.</li> <li>What percentage of racers completed a race between 22 and 25 minutes?</li> </ol>	race time (min)								22.2	22.6	24.9	23.5	22.8	23.3	23.1	21.6	21.3	22.9	25.7	23.3	23.3	22.5	24.4	22.7	24.1	23	22.5	23.2	24.7	24.4	23.3	23.5	23.1	22.5	22.3	22.6	23.6	23.3	23.3	23.4	23.0	23.1	24.5	23.9	20.6	23.5	22.8	24.4
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24.1	23	22.5	23.2	24.7	24.4	23.3	23.5																																										
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23.0	23.1	24.5	23.9	20.6	23.5	22.8	24.4																																										
<p style="text-align: center;"><b>A</b> <i>Students can do</i></p>	<p>The average weight of a sumo wrestler is 325.6 lbs. The range of weight of a sumo wrestler is 200 – 631.4 lbs. Assuming that the weights are normally distributed, sketch a normal curve of sumo wrestler weights. Keeping in mind the 68-95-99.7 rule, estimate what you believe would be one standard deviation in pounds.</p>																																																

**AA8. Statistics ~ AC2 Supplement**

S-IC.1, 2

Learning Targets	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA8b. I can understand and evaluate random processes underlying statistical experiments.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Define population, population parameter, random sample, inference</li> <li><input type="checkbox"/> Choose a probability model for a situation</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Explain why randomization is used to draw a sample that represents a population well</li> <li><input type="checkbox"/> Drawing conclusions based on results from a random sample of a population.</li> <li><input type="checkbox"/> Decide if the data collected is consistent with the selected model or if another model is required.</li> </ul>

<p><b>C</b> <i>Students can do</i></p>	<p>With your team, consider each of the survey questions below. Decide if any bias techniques are being used to influence the survey results. If no bias technique is being used, write "Fair question." Be prepared to explain your thinking to the class.</p> <ol style="list-style-type: none"> <li>a. Jolly Juice has twice the Vitamin E of other brands. Which brand of juice is the healthiest?</li> <li>b. Do you think that people who hurt defenseless animals should be punished?</li> <li>c. Do you agree that Hal Poppington is the best man to be elected Mayor?</li> <li>d. What is your favorite kind of juice?</li> </ol>	<p>A survey was conducted in the following manner: "The Bill of Rights guarantees the right to bear arms so that we can protect our families and our country. Recently, attempts have been made to enact stricter gun controls. Do you want these restrictions?"</p> <p>Another survey was conducted in the following manner: "Last year over 15,000 people were murdered by handguns. That was 68% of all murders. Recently, attempts have been made to enact stricter gun controls. Do you want these restrictions?"</p>
<p><b>B</b> <i>Students can do</i></p>	<p>Can you legitimately conclude from this study that aspirin reduces heart attacks for all men? Why or why not?</p> <p>Can you legitimately conclude from this study that aspirin is linked to reduced heart attacks for all men aged 40 to 84? Why or why not?</p>	<p>Suppose you were conducting a survey to determine what portion of voters in your town support a particular candidate for mayor. Consider each of the following methods for sampling the voting population of your town. State whether each is likely to produce a representative sample and explain your reasoning.</p> <ol style="list-style-type: none"> <li>a. Call one number from each page of the phone book between noon and 2 p.m.</li> <li>b. Survey each person leaving a local grocery store.</li> <li>c. Survey each person leaving a local movie theater.</li> <li>d. Walk around downtown and survey every fourth person you see.</li> </ol>

## A

*Students can do*

Sports announcers frequently get excited when basketball players make several free throw shots in a row. They say things like “he’s on a hot streak tonight!” or “he’s really in the zone—what an amazing performance!”

- Are these “hot” streaks really special, or are they just a natural run to be expected by probability? We will use simulation to determine what really is an unusually long streak of free throws, as opposed to a streak that is expected through normal play. Assume a basketball player has a 50% free throw average, and a typical game has 20 free throw attempts. Use your calculator to randomly generate 20 free throws, with “0” representing a missed shot, and “1” representing making the free throw. Enter  $\text{PRB randInt}(0,1,20)$   $\text{STO} \rightarrow$   $\text{2nd}$   $[L1]$ . In List1 you now have 20 free throws. How long was the longest streak of making free throws (what was the most number of “1”s in a row)?
- Run the simulation 25 more times. Each time record the length of the longest streak.
- How long would a streak have to be before you considered it unusual?

## AA8. Statistics ~ AC2 Supplement

S-IC.3-6

Learning Targets	All of these items are to be covered under this learning target	
	Procedural	Conceptual
AA8c. I can make inferences and justify conclusions from sample surveys, experiments, and observational studies.	<ul style="list-style-type: none"> <li><input type="checkbox"/> Define sample survey, experiment, observational study, and randomization</li> <li><input type="checkbox"/> Outliers</li> </ul>	<ul style="list-style-type: none"> <li><input type="checkbox"/> Describe the difference among sample surveys, experiments, and observational studies</li> <li><input type="checkbox"/> Apply random sampling techniques to draw a sample from a population</li> <li><input type="checkbox"/> Choose appropriate margin of error for sample mean or proportion and use a confidence interval based on the results of the simulation conducted</li> <li><input type="checkbox"/></li> </ul>

<b>C</b> <i>Students can do</i>	<p>A consumer magazine randomly selects 250 of its readers and asks if their luggage was lost on their last airplane flight. Seven out of the 250 lost their luggage. The consumer magazine conducted a simulation to determine the sample-to-sample variability and concluded that about <math>3\% \pm 1.7\%</math> of all passengers lose their luggage. If the magazine had surveyed 1000 readers instead, make a conjecture about what the number of passengers who lose their luggage might be?</p>
<b>B</b> <i>Students can do</i>	<p>Write a question about cell phone use in schools that could be answered using a(n):</p> <p>Sample Survey:</p> <p>Observational Study:</p> <p>Experiment:</p>

# A

Students can do

Students in Miss Hampton's science class tested the effectiveness of detergent in getting dishes clean. They created a gooey paste of hard-to-clean foods (spaghetti sauce, mustard, mashed potatoes, and grape jelly) and smeared 250 clean dinner plates with an exact amount of the food paste. They weighed each plate and randomly placed them into commercial dishwashers. Half the dishwashers had detergent in them, and half had only clean water. After cleaning the dishes, they weighed each plate to determine the portion of food paste that remained. 84% of the food was removed from dishes cleaned with detergent, while only 72% of the food was removed from dishes cleaned without detergent. Using the steps below, explore whether detergent really helps dishes get cleaner than just plain water.

- a. What is the difference in the proportions (detergent minus plain water)? Express your answer as a decimal.
- b. Mrs. Hampton's class ran a computer simulation and determined the sample-to-sample variability of the *difference* between the proportion of food removed by the detergent compared to plain water. They concluded that the *difference* in the true proportion of food removed was  $0.12 \pm 0.085$ .

Is a difference of zero a plausible result considering their margin of error? What does a difference of zero mean in the context of this problem?

- c. Are you convinced that there is a true difference between cleaning with detergent and cleaning with plain water?