R2.7 **Low-birth-weight babies** Researchers in Norway analyzed data on the birth weights of 400,000 newborns over a 6-year period. The distribution of birth weights is Normal with a mean of 3668 grams and a standard deviation of 511 grams.[16] Babies that weigh less than 2500 grams at birth are classified as "low birth weight."

(a) What percent of babies will be identified as low birth weight? Show your work.

(b) Find the quartiles of the birth weight distribution. Show your work.

R2.8 **Ketchup** A fast-food restaurant has just installed a new automatic ketchup dispenser for use in preparing its burgers. The amount of ketchup dispensed by the machine follows a Normal distribution with mean 1.05 ounces and standard deviation 0.08 ounce.

(a) If the restaurant's goal is to put between 1 and 1.2 ounces of ketchup on each burger, what percent of the time will this happen? Show your work.

(b) Suppose that the manager adjusts the machine's settings so that the mean amount of ketchup dispensed is 1.1 ounces. How much does the machine's standard deviation have to be reduced to ensure that at least 99% of the restaurant's burgers have between 1 and 1.2 ounces of ketchup on them?

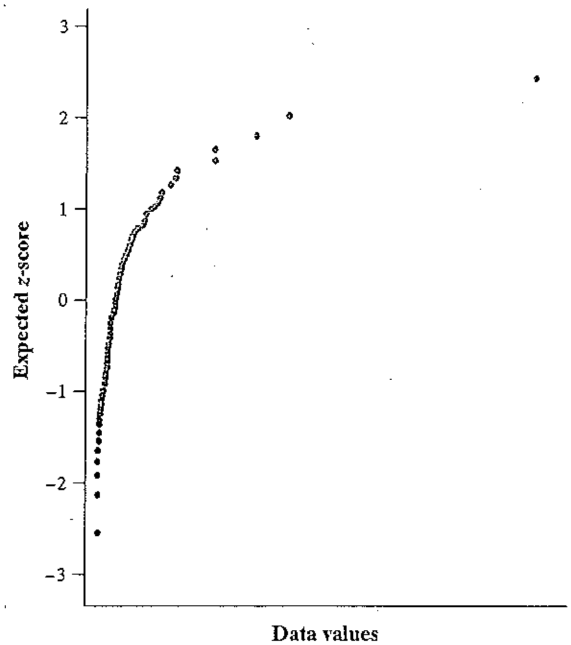
R2.9 **Grading managers** Many companies "grade on a bell curve" to compare the performance of their managers and professional workers. This forces the use of some low performance ratings, so that not all workers are listed as "above average." Ford Motor Company's "performance management process" for a time assigned 10% A grades, 80% B grades, and 10% C grades to the company's 18,000 managers. Suppose that Ford's performance scores really are Normally distributed. This year, managers with scores less than 25 received C's, and those with scores above 475 received A's. What are the mean and standard deviation of the scores? Show your work.

R2.10 **Fruit fly thorax lengths** Here are the lengths in millimeters of the thorax for 49 male fruit flies:[17]

0.64	0.64	0.64	0.68	0.68	0.68	0.72	0.72	0.72	0.72	0.74	0.76	0.76
0.76	0.76	0.76	0.76	0.76	0.76	0.78	0.80	0.80	0.80	0.80	0.80	0.82
0.82	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.88	0.88
0.88	0.88	0.88	0.88	0.88	0.88	0.92	0.92	0.92	0.94			

Are these data approximately Normally distributed? Give appropriate graphical and numerical evidence to support your answer.

R2.11 **Assessing Normality** A Normal probability plot of a set of data is shown here. Would you say that these measurements are approximately Normally distributed? Why or why not?



Chapter 2 AP® Statistics Practice Test

Section I: Multiple Choice *Select the best answer for each question.*

T2.1 Many professional schools require applicants to take a standardized test. Suppose that 1000 students take such a test. Several weeks after the test, Pete receives his score report: he got a 63, which placed him at the 73rd percentile. This means that

(a) Pete's score was below the median.

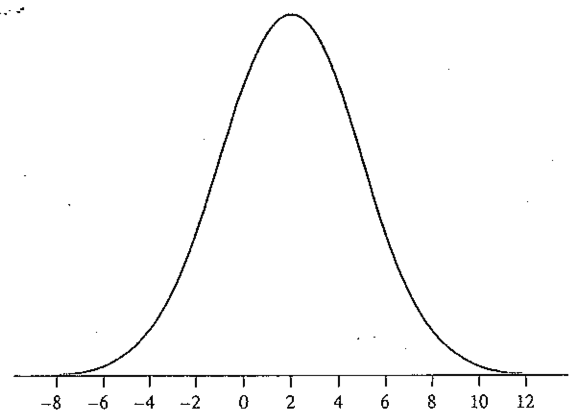
(b) Pete did worse than about 63% of the test takers.

(c) Pete did worse than about 73% of the test takers.

(d) Pete did better than about 63% of the test takers.

(e) Pete did better than about 73% of the test takers.

T2.2 For the Normal distribution shown, the standard deviation is closest to

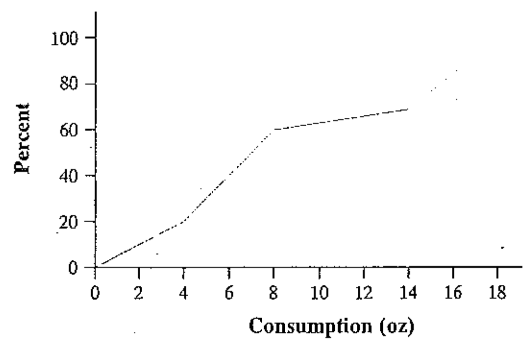


(a) 0 (b) 1 (c) 2 (d) 3 (e) 5

T2.3 Rainwater was collected in water collectors at 30 different sites near an industrial complex, and the amount of acidity (pH level) was measured. The mean and standard deviation of the values are 4.60 and 1.10, respectively. When the pH meter was recalibrated back at the laboratory, it was found to be in error. The error can be corrected by adding 0.1 pH units to all of the values and then multiplying the result by 1.2. The mean and standard deviation of the corrected pH measurements are

(a) 5.64, 1.44
(b) 5.64, 1.32
(c) 5.40, 1.44
(d) 5.40, 1.32
(e) 5.64, 1.20

T2.4 The figure shows a cumulative relative frequency graph of the number of ounces of alcohol consumed per week in a sample of 150 adults who report drinking alcohol occasionally. About what percent of these adults consume between 4 and 8 ounces per week?

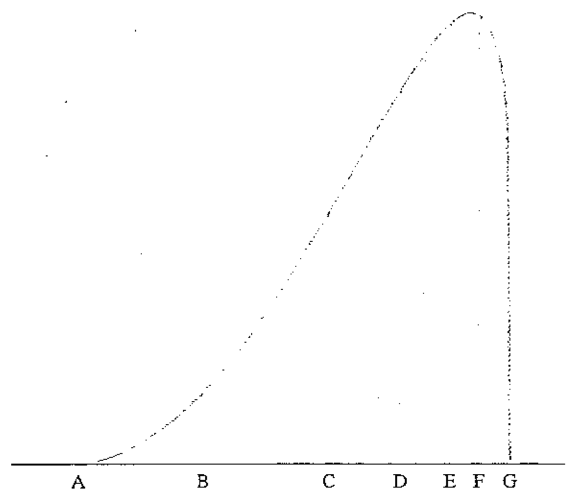


(a) 20% (b) 40% (c) 50% (d) 60% (e) 80%

T2.5 The average yearly snowfall in Chillyville is Normally distributed with a mean of 55 inches. If the snowfall in Chillyville exceeds 60 inches in 15% of the years, what is the standard deviation?

(a) 4.83 inches
(b) 5.18 inches
(c) 6.04 inches
(d) 8.93 inches
(e) The standard deviation cannot be computed from the given information.

T2.6 The figure shown is the density curve of a distribution. Seven values are marked on the density curve. Which of the following statements is true?



(a) The mean of the distribution is E.
(b) The area between B and F is 0.50.
(c) The median of the distribution is C.
(d) The 3rd quartile of the distribution is D.
(e) The area between A and G is 1.

T2.7 If the heights of a population of men follow a Normal distribution, and the middle 99.7% have heights between 5′0″ and 7′0″, what is your estimate of the standard deviation of the heights in this population?

(a) 1″ (b) 3″ (c) 4″ (d) 6″ (e) 12″

T2.8 Which of the following is *not* correct about a standard Normal distribution?

(a) The proportion of scores that satisfy $0 < z < 1.5$ is 0.4332.
(b) The proportion of scores that satisfy $z < -1.0$ is 0.1587.
(c) The proportion of scores that satisfy $z > 2.0$ is 0.0228.
(d) The proportion of scores that satisfy $z < 1.5$ is 0.9332.
(e) The proportion of scores that satisfy $z > -3.0$ is 0.9938.

Questions T2.9 and T2.10 refer to the following setting. Until the scale was changed in 1995, SAT scores were based on a scale set many years ago. For Math scores, the mean under the old scale in the 1990s was 470 and the standard deviation was 110. In 2013, the mean was 515 and the standard deviation was 116.

T2.9 What is the standardized score (z-score) for a student who scored 500 on the old SAT scale?

(a) −30 (b) −0.27 (c) −0.13 (d) 0.13 (e) 0.27

T2.10 Gina took the SAT in 1994 and scored 500. Her cousin Colleen took the SAT in 2013 and scored 530. Who did better on the exam, and how can you tell?

(a) Colleen—she scored 30 points higher than Gina.

(b) Colleen—her standardized score is higher than Gina's.

(c) Gina—her standardized score is higher than Colleen's.

(d) Gina—the standard deviation was bigger in 2013.

(e) The two cousins did equally well—their z-scores are the same.

Section II: Free Response *Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.*

T2.11 As part of the President's Challenge, students can attempt to earn the Presidential Physical Fitness Award or the National Physical Fitness Award by meeting qualifying standards in five events: curl-ups, shuttle run, sit and reach, one-mile run, and pull-ups. The qualifying standards are based on the 1985 School Population Fitness Survey. For the Presidential Award, the standard for each event is the 85th percentile of the results for a specific age group and gender among students who participated in the 1985 survey. For the National Award, the standard is the 50th percentile. To win either award, a student must meet the qualifying standard for all five events.

Jane, who is 9 years old, did 40 curl-ups in one minute. Matt, who is 12 years old, also did 40 curl-ups in one minute. The qualifying standard for the Presidential Award is 39 curl-ups for Jane and 50 curl-ups for Matt. For the National Award, the standards are 30 and 40, respectively.

(a) Compare Jane's and Matt's performances using percentiles. Explain in language simple enough for someone who knows little statistics to understand.

(b) Who has the higher standardized score (z-score), Jane or Matt? Justify your answer.

T2.12 The army reports that the distribution of head circumference among male soldiers is approximately Normal with mean 22.8 inches and standard deviation 1.1 inches.

(a) A male soldier whose head circumference is 23.9 inches would be at what percentile? Show your method clearly.

(b) The army's helmet supplier regularly stocks helmets that fit male soldiers with head circumferences between 20 and 26 inches. Anyone with a head circumference outside that interval requires a customized helmet order. What percent of male soldiers require custom helmets?

(c) Find the interquartile range for the distribution of head circumference among male soldiers.

T2.13 A study recorded the amount of oil recovered from the 64 wells in an oil field. Here are descriptive statistics for that set of data from Minitab.

Descriptive Statistics: Oilprod

Variable	n	Mean	Median	StDev	Min	Max	Q_1	Q_3
Oilprod	64	48.25	37.80	40.24	2.00	204.90	21.40	60.75

Does the amount of oil recovered from all wells in this field seem to follow a Normal distribution? Give appropriate statistical evidence to support your answer.

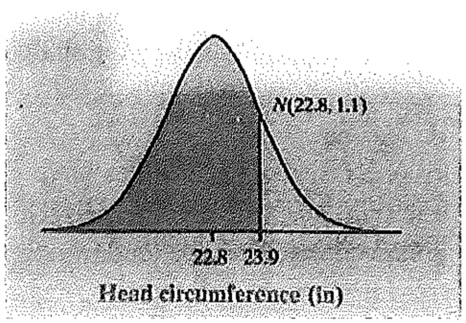
Answers to Ch 2 AP Statistics Practice TEST

MULTIPLE CHOICE

1. e
2. d
3. b
4. b
5. a
6. e
7. c
8. e
9. e
10. c

FREE RESPONSE

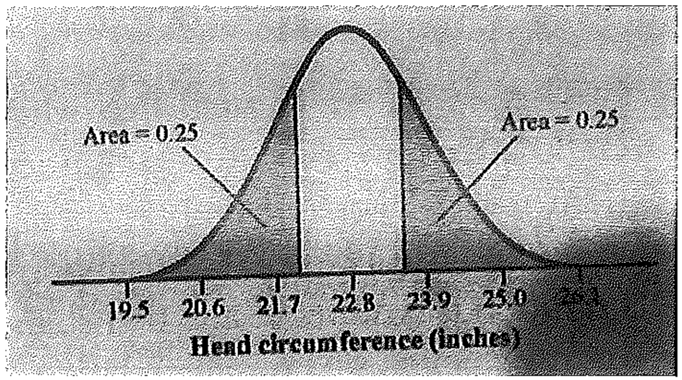
11. **(a)** Jane's performance was better. Her performance (40) exceeded the standard for the Presidential award (39), she performed above the 85^{th} percentile. Matt's performance (40) met the standard for the National award (40), meaning he performed at the 50^{th} percentile. **(b)** Because Jane's score has a higher percentile than Matt's score, she is farther to the right on her distribution than Matt is in his. Therefore, Jane's standardized score will likely be greater than Matt's.
12. **(a)** For males soldiers, head circumference follows an N(22.8, 1.1) distribution and we want to find the percent of soldiers with head circumference less than 23.9 inches.



z= (23.9-22.8)/1.1 = 1. From Table A, the proportion of z-scores below 1 is 0.8413. Using technology: *normalcdf(lower: -1000, upper: 23.9, mean: 22.8, standard deviation: 1.1)* = 0.8413. About 84% of soldiers have head circumferences less than 23.9 inches. Thus 23.9 inches is at the 84^{th} percentile.

(b) For male soldiers, head circumference follows an N(22.8. 1.1) distribution and we want to find the percent of soldiers with head circumferences less than 20 inches or greater than 26 inches. Z = (20-22.8)/1.1 = -2.55 and z = (26-22.8)/1.1 = 2.91. From Table A, the proportion of z-scores below z=-2.55 is 0.0054 and the proportion of z-scores above 2.91 is 1-0.9982 = 0.0018, for a total of 0.0054+0.0018=0.0072. Using technology, 1 – *normalcdf(20, 26, 22.8, 1.1)* = 1 – 0.9927 = 0.0073. A little less than 1% of soldiers have head circumferences less than 20 inches or greater than 26 inches and require custom helmets.

(c) For male soldiers, head circumference follows an N(22.8, 1.1) distribution. The first quartile is the boundary value with 25% of the area to its left. The third quartile is the boundary value with 75% of the area to its left. See graph below. A z-score of -0.67 gives the value closest to 0.25 (0.2514). Solving -0.67 = (x-22.8)/1.1 gives Q1 = 22.063. A z-score of 0.67 gives the value closest to 0.75(0.7486). Solving 0.67 = (x-22.8)/1.1 gives Q3 = 23.537. Using technology, *invNorm(area: 0.25, mean: 22.8, standard deviation: 1.1)* gives Q1 = 22.058 and *invNorm(0.75, 22.8, 1.1)* gives Q3 = 23.542. Thus, IQR = 23.542-22.058 = 1.484 inches.



13. No, First, there is a large difference between the mean and median. In a Normal distribution the mean and median are the same, but in this distribution the mean is 48.25 and the median is 37.80. Second, the distance between the minimum and the median is 35.80 but the distance between the median and the maximum is 167.10. In a normal distribution, these distances should be about the same (because a Normal distribution is symmetric). Both of these facts suggest that the distribution is skewed to the right.

Chapter 2 FRAPPY!

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

The distribution of scores on a recent test closely followed a Normal distribution with a mean of 22 points and a standard deviation of 4 points.

(a) What proportion of the students scored at least 25 points on this test?

(b) What is the 31st percentile of the distribution of test scores?

(c) The teacher wants to transform the test scores so that they have an approximately Normal distribution with a mean of 80 points and a standard deviation of 10 points. To do this, she will use a formula in the form:

$$new\ score = a + b\ (old\ score)$$

Find the values of a and b that the teacher should use to transform the distribution of test scores.

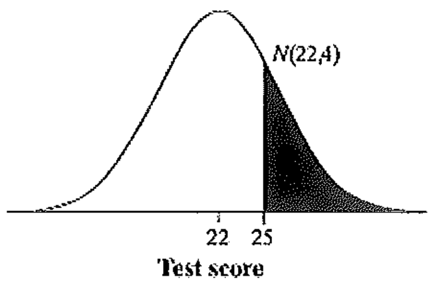
(d) Before the test, the teacher gave a review assignment for homework. The maximum score on the assignment was 10 points. The distribution of scores on this assignment had a mean of 9.2 points and a standard deviation of 2.1 points. Would it be appropriate to use a Normal distribution to calculate the proportion of students who scored below 7 points on this assignment? Explain.

Chapter 2 FRAPPY!
Scoring Guidelines

Intent of the question The primary goals of this question are to assess a student's ability to: (1) use a Normal distribution to calculate the proportion of observations in a specified region; (2) use a Normal distribution to determine a specified percentile; (3) determine a linear transformation from summary statistics; (4) justify if a distribution of data is approximately Normal based on summary statistics.

Model Solution

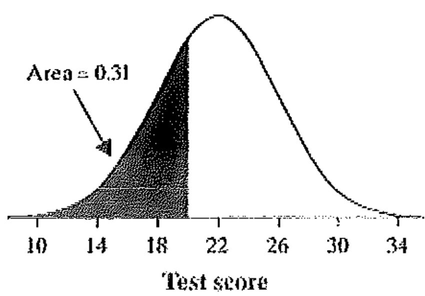
(a) Step 1: State the distribution and the values of interest. The distribution of scores follows a Normal distribution with mean 22 and standard deviation 4. We want to find the proportion of scores that are at least 25.



Step 2: Perform calculations--show your work. The standardized score for the boundary value is $z = \frac{25-22}{4} = 0.75$. From Table A, the proportion of z-scores above 0.75 is 1 – 0.7734 = 0:2266. *Using technology:* The command normalcdf(lower: 25, upper: 1000, μ: 22, σ: 4) gives an area of 0.2266.

Step 3: Answer the question. The proportion of students who scored at least 25 points is about 0.23.

(b) Step 1: State the distribution and the values of interest. The distribution of scores follows a Normal distribution with mean 22 and standard deviation 4. The 31st percentile is the boundary value x with 31% of the distribution to its left.



Step 2: Perform calculations--show your work. Look in the body of Table A for a value closest to 0.31. A z-score of –0.50 gives the closest value (0.3085). Solving $-0.50 = \frac{x-22}{4}$ gives x = 20. *Using technology:* The command invNorm(area: 0.31, μ: 22, σ: 4) gives a value of 20.02.

Step 3: Answer the question. The 31st percentile is about 20 points.

(c) Because adding a constant does not affect measures of spread:

new SD = b (old SD) → 10 = b (4) → b = 2.5

Because adding a constant and multiplying by a constant affect measures of center:

new mean = $a + b$ (old mean) → 80 = a + 2.5 (22) → a = 25

The linear transformation should be: *new score* = 25 + 2.5 (*old score*)

(d) If the distribution was Normal, scores should go at least 2 SD above the mean. However, in this distribution, the highest possible score is only $z = (10 - 9.2)/2.1 = 0.38$ standard deviations above the mean. Therefore, it would be inappropriate to use a Normal distribution to do calculations.

Scoring
Parts (a)–(d) are scored essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows

Essentially correct (E) if the response correctly includes the following three components:
1. Indicates use of a normal distribution and clearly identifies the correct parameter values (using a z-score is sufficient);
2. Uses the correct boundary value;
3. Reports the correct normal proportion consistent with components 1 and 2.

Partially correct (P) if the response correctly includes two of the three components listed above.

Incorrect (I) if the response does not satisfy the criteria for an E or a P.

Note: The following are examples of clearly identified parameters for component 1:
- Writes $\mu = 22$ and $\sigma = 4$
- Explicitly labels the mean and standard deviation in a normalcdf calculator statement.
- Sketches a normal curve, labels 22 as the mean, and labels two additional consecutive values separated by 4.

Part (b) is scored as follows

Essentially correct (E) if the response correctly includes the following three components:

1. Indicates use of a normal distribution and clearly identifies the correct parameter values (using a z-score is sufficient);
2. Uses the correct area value;
3. Reports the correct percentile consistent with components 1 and 2.

Partially correct (P) if the response correctly includes two of the three components listed above.

Incorrect (I) if the response does not satisfy the criteria for an E or a P.

Note: The following are examples of clearly identified parameters for component 1:

- Writes $\mu = 22$ and $\sigma = 4$
- Explicitly labels the mean and standard deviation in an invNorm calculator statement.
- Sketches a normal curve, labels 22 as the mean, and labels two additional consecutive values separated by 4.
- Correct identification of parameters in part (a)

Part (c) is scored as follows

Essentially correct (E) if the response calculates both values correctly and shows appropriate work.

Partially correct (P) if the response calculates one of the two values correctly and shows appropriate work.

OR

Calculates both values correctly but does not show appropriate work.

Incorrect (I) otherwise

Note: If the standard deviation is calculated incorrectly and the mean is calculated correctly based on the incorrect standard deviation with appropriate work shown, score the response partially correct (P).

Part (d) is scored as follows

Essentially correct (E) if the response includes the following 3 components

1. States a relevant characteristic of a Normal distribution
2. Uses all three summary statistics (mean, SD, max) to show that the distribution of scores does not share the characteristic of the Normal distribution described in component 1
3. Concludes that the use of a Normal distribution would not be appropriate.

Partially correct (P) if the response includes 2 of the 3 components

Incorrect (I) otherwise.

Notes:

- A response that calculates that approximately 35% of scores should be above 10 in a Normal distribution with $\mu = 9.2$ and $\sigma = 2.1$, states that 0% will be above 10 in the actual distribution, and concludes that a Normal distribution would be inappropriate should be scored essentially correct (E).
- A response that states that it would be inappropriate to use a Normal distribution because the maximum score of 10 is less than 1 standard deviation above the mean should be scored partially correct (P) because it doesn't state a relevant characteristic of a Normal distribution (e.g., "In a Normal distribution, the max should be 2 or 3 standard deviations above the mean.")
- A response that states that it would be inappropriate to use a Normal distribution because 68% of the values should be within 1 standard deviation of the mean (or 95% should be within 2 standard deviations of the mean) should be scored partially correct (P) because the response doesn't show how the actual distribution doesn't share this characteristic.

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as ½ point. If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication, particularly in parts (a) and (d).

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response