KEY

Chapter 6 AP® Statistics Practice Test

Section I: Multiple Choice *Select the best answer for each question.*

Questions T6.1 to T6.3 refer to the following setting. A psychologist studied the number of puzzles that subjects were able to solve in a five-minute period while listening to soothing music. Let X be the number of puzzles completed successfully by a randomly chosen subject. The psychologist found that X had the following probability distribution:

Value:	1	2	3	4
Probability:	0.2	0.4	0.3	0.1

T6.1 What is the probability that a randomly chosen subject completes more than the expected number of puzzles in the five-minute period while listening to soothing music?

(a) 0.1
(b) 0.4
(c) 0.8
(d) 1
(e) Cannot be determined

$E(X) = 1(.2) + 2(.4) + 3(.3) + 4(.1) = 2.3$
$P(X > 2.3) = .3 + .1 = .4$

T6.2 The standard deviation of X is 0.9. Which of the following is the best interpretation of this value?

(a) About 90% of subjects solved 3 or fewer puzzles.
(b) About 68% of subjects solved between 0.9 puzzles less and 0.9 puzzles more than the mean.
(c) The typical subject solved an average of 0.9 puzzles.
(d) The number of puzzles solved by subjects typically differed from the mean by about 0.9 puzzles.
(e) The number of puzzles solved by subjects typically differed from one another by about 0.9 puzzles.

T6.3 Let D be the difference in the number of puzzles solved by two randomly selected subjects in a five-minute period. What is the standard deviation of D?

$\sigma = .9$ $\quad D = X_1 - X_2$ $\quad \sigma^2 = \sqrt{.9^2 + .9^2} = 1.27$

(a) 0 (b) 0.81 (c) 0.9 (d) 1.27 (e) 1.8

T6.4 Suppose a student is randomly selected from your school. Which of the following pairs of random variables are most likely independent?

(a) X = student's height; Y = student's weight
(b) X = student's IQ; Y = student's GPA
(c) X = student's PSAT Math score; Y = student's PSAT Verbal score
(d) X = average amount of homework the student does per night; Y = student's GPA
(e) X = average amount of homework the student does per night; Y = student's height

T6.5 A certain vending machine offers 20-ounce bottles of soda for \$1.50. The number of bottles X bought from the machine on any day is a random variable with mean 50 and standard deviation 15. Let the random variable Y equal the total revenue from this machine on a given day. Assume that the machine works properly and that no sodas are stolen from the machine. What are the mean and standard deviation of Y?

(a) $\mu_Y = \$1.50$, $\sigma_Y = \$22.50$ $\quad \mu_X = 50$
(b) $\mu_Y = \$1.50$, $\sigma_Y = \$33.75$ $\quad \sigma_X = 15$
(c) $\mu_Y = \$75$, $\sigma_Y = \$18.37$
(d) $\mu_Y = \$75$, $\sigma_Y = \$22.50$ $\quad \mu_Y = 1.50(50) = 75$
(e) $\mu_Y = \$75$, $\sigma_Y = \$33.75$ $\quad \sigma_Y = 1.50(15) = 22.5$

T6.6 The weight of tomatoes chosen at random from a bin at the farmer's market follows a Normal distribution with mean $\mu = 10$ ounces and standard deviation $\sigma = 1$ ounce. Suppose we pick four tomatoes at random from the bin and find their total weight T. The random variable T is

$T = X_1 + X_2 + X_3 + X_4$

(a) Normal, with mean 10 ounces and standard deviation 1 ounce.
$\mu_T = \mu_1 + \mu_2 + \mu_3 + \mu_4$
(b) Normal, with mean 40 ounces and standard deviation 2 ounces.
$\sigma_T = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2} = 2$
(c) Normal, with mean 40 ounces and standard deviation 4 ounces.
(d) binomial, with mean 40 ounces and standard deviation 2 ounces.
(e) binomial, with mean 40 ounces and standard deviation 4 ounces.

T6.7 Which of the following random variables is geometric?

(a) The number of times I have to roll a die to get two 6s.
(b) The number of cards I deal from a well-shuffled deck of 52 cards until I get a heart. P changes!! w/o replace
(c) The number of digits I read in a randomly selected row of the random digits table until I find a 7. ←
(d) The number of 7s in a row of 40 random digits. X
(e) The number of 6s I get if I roll a die 10 times. X

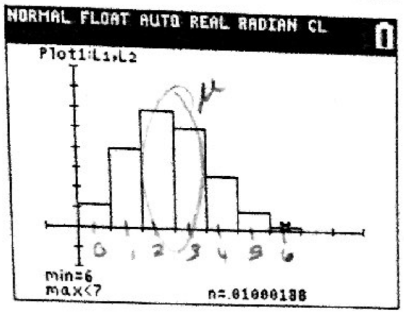
T6.8 Seventeen people have been exposed to a particular disease. Each one independently has a 40% chance of contracting the disease. A hospital has the capacity to handle 10 cases of the disease. What is the probability that the hospital's capacity will be exceeded?

(a) 0.011
(b) 0.035
(c) 0.092
(d) 0.965 $\quad$ 1, 2, 3, 4 ... 10, 11, 12, 13, ...
(e) 0.989

$P(X \geq 11) = 1 - P(X \leq 10) = 1 - \text{binomcdf}(17, .4, 10)$
$= .0348$

2

T6.9 The figure shows the probability distribution of a discrete random variable X. Note that the cursor is on the histogram bar representing a value of 6. Which of the following best describes this random variable?



(a) Binomial with $n = 8, p = 0.1$
(b) Binomial with $n = 8, p = 0.3$
(c) Binomial with $n = 8, p = 0.8$
(d) Geometric with $p = 0.1$
(e) Geometric with $p = 0.2$

$\mu = np$

$\frac{3}{8} = \frac{8p}{8}$

$p = .375$

T6.10 A test for extrasensory perception (ESP) involves asking a person to tell which of 5 shapes—a circle, star, triangle, diamond, or heart—appears on a hidden computer screen. On each trial, the computer is equally likely to select any of the 5 shapes. Suppose researchers are testing a person who does not have ESP and so is just guessing on each trial. What is the probability that the person guesses the first 4 shapes incorrectly but gets the fifth correct?

(a) 1/5

(b) $\left(\frac{4}{5}\right)^4$

(c) $\left(\frac{4}{5}\right)^4 \cdot \left(\frac{1}{5}\right)$

(d) $\binom{5}{1} \cdot \left(\frac{4}{5}\right)^4 \cdot \left(\frac{1}{5}\right)$

(e) 4/5

Section II: Free Response *Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.*

T6.11 Let Y denote the number of broken eggs in a randomly selected carton of one dozen "store brand" eggs at a local supermarket. Suppose that the probability distribution of Y is as follows.

Value y_i:	0	1	2	3	4
Probability p_i:	0.78	0.11	0.07	0.03	0.01

(a) What is the probability that at least 10 eggs in a randomly selected carton are *unbroken*?

(b) Calculate and interpret μ_Y.

(c) Calculate and interpret σ_Y. Show your work.

(d) A quality control inspector at the store keeps looking at randomly selected cartons of eggs until he finds one with at least 2 broken eggs. Find the probability that this happens in one of the first three cartons he inspects.

Geom

T6.12 *Ladies Home Journal* magazine reported that 66% of all dog owners greet their dog before greeting their spouse or children when they return home at the end of the workday. Assume that this claim is true. Suppose 12 dog owners are selected at random. Let X = the number of owners who greet their dogs first.

(a) Explain why it is reasonable to use the binomial distribution for probability calculations involving X.

(b) Only 4 of the owners in the random sample greeted their dogs first. Does this give convincing evidence against the *Ladies Home Journal* claim? Calculate an appropriate probability to support your answer.

T6.13 Ed and Adelaide attend the same high school, but are in different math classes. The time E that it takes Ed to do his math homework follows a Normal distribution with mean 25 minutes and standard deviation 5 minutes. Adelaide's math homework time A follows a Normal distribution with mean 50 minutes and standard deviation 10 minutes. Assume that E and A are independent random variables.

(a) Randomly select one math assignment of Ed's and one math assignment of Adelaide's. Let the random variable D be the difference in the amount of time each student spent on their assignments: $D = A - E$. Find the mean and the standard deviation of D. Show your work.

(b) Find the probability that Ed spent longer on his assignment than Adelaide did on hers. Show your work.

T6.14 According to the Census Bureau, 13% of American adults (aged 18 and over) are Hispanic. An opinion poll plans to contact an SRS of 1200 adults.

(a) What is the mean number of Hispanics in such samples? What is the standard deviation?

(b) Should we be suspicious if the sample selected for the opinion poll contains 15% Hispanic people? Compute an appropriate probability to support your answer.

.11

not   2 broken

1 2 3 4 5 6 7 8 9 10 11 12

a) $P(\geq 10 \text{ eggs unbroken}) = P(Y \leq 2)$

$P(Y \leq 2) = P(Y=0) + P(Y=1) + P(Y=2)$

$= .78 + .11 + .07 = .96$

There is about a 96% chance that at least 10 eggs are not broken in a randomly selected carton of eggs

b) $\mu_Y = 0(.78) + 1(.11) + 2(.07) + 3(.03) + 4(.01) = .38$

After selecting many, many cartons of one dozen eggs, the average number of broken eggs is about .38

c) $\sigma_Y = \sigma_Y^2 = \sum (y_i - \mu_x)^2 p_i$

$= (0-.38)^2(.78) + (1-.38)^2 \cdot (.11) + \ldots$

$\sigma_Y = .62194$

The typical variation from the mean (.38) is about .62194 after many, many trials.

d) B I N S ?

B: Success: broken eggs, failure: not broken eggs

I: reasonable, p of one carton doesn't tell us anything about p of the next carton

N: until 1 out of 3 are broken

S: p = .38 for every trial

not broken   broken

0 1 2 3 4

Y=1: B^c
Y=2: BB^c
Y=3: BBB^c

$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4) = .11$

Let Y = # of cartons until (and including) the 1st carton w/ at least 2 broken

$P(Y \leq 3) = P(Y=1) + P(Y=2) + P(Y=3) = (.11) + (.89)(.11) + (.89)^2(.11) = .2950$

or geometcdf(p=.11, x=3) = .2950

The prob. that the QA inspector finds at least 2 broken eggs is about 30%.

6.12

a) B : success: greet dog first, failure: does not greet dog first

I : reasonable to assume that the outcome of one person's greeting does not tell us anything about the next person's greeting

N : fixed at $n=12$

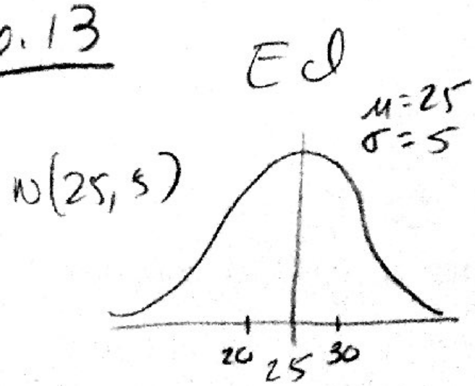
S : $p=.66$ for each person

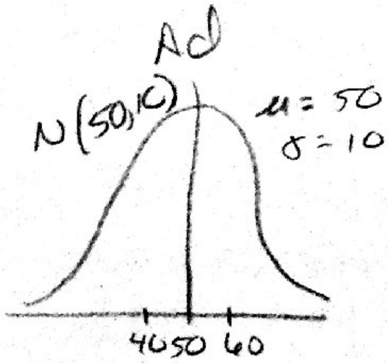
b) $P(X \leq 4) = \text{binomcdf}(n=12, p=.66, k=4)$

$= .0213$

The prob. of only 4 four out of the 12 owners greeting their dogs first is only ~ 2 %. Yes, this does give convincing evidence against the magazine since the probability is so small.

6.13





a) $\mu_E = 25, \sigma_E = 5$ $\qquad$ $\mu_a = 50, \sigma_a = 10$

$D = A - E$

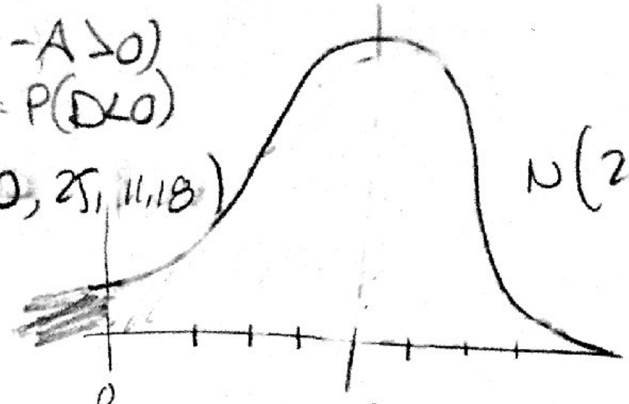
$\mu_D = \mu_A - \mu_E = 50 - 25 = \boxed{25 \text{ min}}$

$\sigma_D^2 = \sigma_A^2 + \sigma_E^2 = 10^2 + 5^2 = 75$

$\sigma_D = \sqrt{125} \approx \boxed{11.18 \text{ min}}$

c) $P(E > A) = P(E - A > 0)$
$= P(-D > 0) = P(D < 0)$

$=$ normalcdf$(-10000, 0, 25, 11.18)$



$N(25, 11.18)$

$= .012671$

The prob. that Ed spent longer on his assignment than Adelaine is about 1.3%.

6.14

a) $\mu_X = (.13)(1200) = 156$

B I N S

$\sigma_X = \sqrt{np(1-p)}$

$= \sqrt{1200(.13)(.87)}$

$= 11.6498927$

b) 15% of $1200 = 180$

$P(X \geq 180) = 1 - P(X \leq 179) = 1 -$ binomcdf $(n = 1200, p = .13, k = 179)$

$= .023461$

$= \sim 2.3\%$

Yes, we should be suspicious if the sample selected for the opinion poll contains 15% because the probability that would occur is only 2.3% which means it's unlikely to happen.