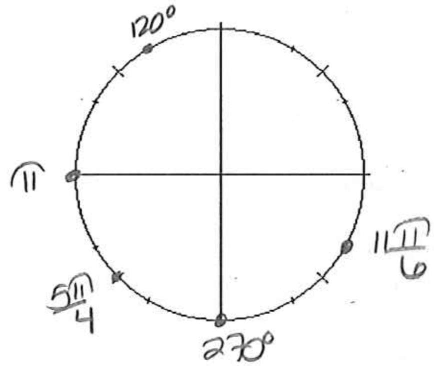
PreCalc
SPRING 2017

Name KEY
Date ______ Pd 7

PRACTICE MIDTERM I
NO CALCULATOR

1. Mark and label the location on the given unit circle where the terminal side of each angle listed below intersects it.

$\pi, 120°, \frac{11\pi}{6}, \frac{5\pi}{4}, 270°$



2. Write the exact coordinates of the point on the unit circle for each angle in the space provided below. Fill in missing degree and radian measures.

Radian Measure	Degree Measure	Coordinates
π	$180°$	$(-1, 0)$
$\frac{2\pi}{3}$	120°	$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{7\pi}{6}$	$210°$	$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\frac{5\pi}{4}$	$225°$	$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\frac{3\pi}{2}$	270°	$(0, -1)$

3. Find the exact value. Show all work.

a) $\cos\left(\frac{14\pi}{3}\right)$

$\frac{14\pi}{3} - \underbrace{\frac{6\pi}{3}}_{2\pi} = \frac{8\pi}{3} - \underbrace{\frac{6\pi}{3}}_{2\pi} = \frac{2\pi}{3}$

$\cos\left(\frac{14\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

b) $\csc\left(-\frac{3\pi}{4}\right) = \frac{1}{\sin\left(-\frac{3\pi}{4}\right)} = \frac{1}{-\sin\left(\frac{3\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}}$

$= -\frac{2}{\sqrt{2}} \cdot \underbrace{\frac{\sqrt{2}}{\sqrt{2}}}_{\text{rationalize}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

4. Find the exact value of the following trigonometric expression. Write as a single term. Show all work.

$\sec\theta = \frac{1}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$$\frac{1}{\sec^2\left(\frac{\pi}{3}\right)} + \frac{1}{\csc^2\left(\frac{\pi}{3}\right)}$$

$$= \frac{1}{\frac{1}{\cos^2\left(\frac{\pi}{3}\right)}} + \frac{1}{\frac{1}{\sin^2\left(\frac{\pi}{3}\right)}}$$

$$= \frac{1}{\frac{1}{\left(\frac{1}{2}\right)^2}} + \frac{1}{\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}} = \frac{1}{1/4} + \frac{1}{1/\frac{3}{4}}$$

$$= \frac{1}{4} + \frac{1}{\frac{4}{3}}$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = \boxed{1}$$

5. Find the value of θ on the interval $\left[0, \frac{\pi}{2}\right]$. Use exact form. Show all work.

a) $csc(\theta) = 2$

$\frac{1}{\sin\theta} = \frac{2}{1}$

$\frac{\cancel{2}\sin\theta}{\cancel{2}} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}$

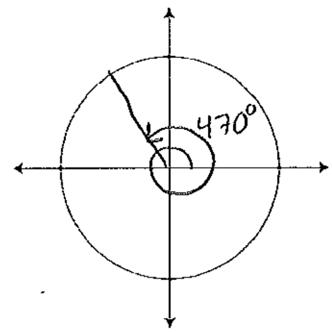
b) $\tan(\theta) = 1$

$\frac{\sin\theta}{\cos\theta} = 1$

$\frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

$\theta = \pi/4$

6. Illustrate the location of angle $\theta = 470°$ on the unit circle provided.



a) What is the measure of θ in radians? Leave your answer in terms of π, in reduced form. Show all work.

$470° \cdot \frac{\pi}{180°} = \frac{47\pi}{18}$

b) Find an angle co-terminal with θ that is between $0°$ *and* $360°$.

$110°$

$470° - 360° = 110°$

7. A point on a circle, *P*, is (-1, -6). Find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, and sec (θ). Use exact form. Show all work.

$(-1)^2 + (-6)^2 = r^2$

$37 = r^2$

$r = \sqrt{37}$

$\sin\theta = \frac{y}{r} = \frac{-6}{\sqrt{37}} \cdot \frac{\sqrt{37}}{\sqrt{37}} = \frac{-6\sqrt{37}}{37}$

$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{37}} \cdot \frac{\sqrt{37}}{\sqrt{37}} = \frac{-\sqrt{37}}{37}$

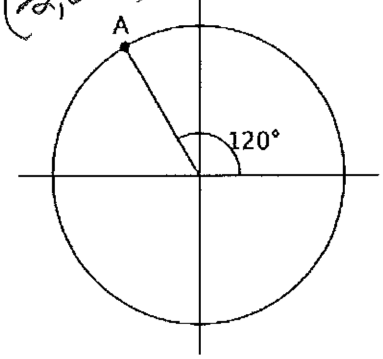
$\sec\theta = \frac{r}{x} = \frac{\sqrt{37}}{-1}$

$= -\sqrt{37}$

$\tan\theta = \frac{y}{x} = \frac{-6}{-1} = 6$

8. Given a circle with radius 4 and the given angle measure; find the exact value of the coordinates of the point A. Show all work.

$(-2, 2\sqrt{3})$



$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$

$\cos(120°) = \frac{x}{4}$ $\sin\theta = \frac{y}{4}$

$x = 4\cos(120°)$ $\sin(120°) = \frac{y}{4}$

$x = 4\left(-\frac{1}{2}\right) = -2$ $\frac{\sqrt{3}}{2} = \frac{y}{4}$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{4\sqrt{3}}{2}$

$y = 2\sqrt{3}$

9. If $\sin(\theta) = \frac{1}{3}$ and $\frac{\pi}{2} \leq \theta \leq \pi$, find the exact values of: (Show all work).

QII

a) $\cos(\theta)$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$

$\frac{1}{9} + \cos^2\theta = 1$

$\cos^2\theta = \frac{9}{9} - \frac{1}{9}$

$\cos^2\theta = \frac{8}{9}$

$\cos\theta = -\sqrt{8/9}$

$\cos\theta = \frac{-\sqrt{8}}{\sqrt{9}}$

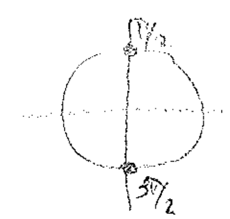
$\cos\theta = -\frac{2\sqrt{2}}{3}$

b) $\cot(\theta)$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-\frac{2\sqrt{2}}{\cancel{3}}}{\frac{1}{\cancel{3}}}$

$= -2\sqrt{2}$

(0, 1)



10. Answer True or False.

F a) The period of $y = 5\cos(\theta) + 3$ *is* π

T b) The domain of $y = \sin(x)$ *is* $(-\infty, \infty)$

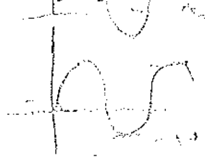
F c) On an interval of length 2π, the function $y = 5\cos(3x + 4) - 8$ will complete 5 cycles.

F d) If the $\cos(\theta) > 0$ *and* $\tan(\theta) < 0$ *then* θ is in Quadrant II.

$\tan(x) = \frac{\sin(x)}{\cos(x)}$

T e) The graph of the function $y = \tan(x)$ will have a vertical asymptote at $x = \frac{\pi}{2}$

F f) The maximum y-value of $y = 10\cos(x) + 25$ *is* 10.



F g) $\cos(-\pi) = -\cos(\pi)$.

11. State the amplitude, period, horizontal shift, and phase shift of $f(x) = 4\sin\left(2x + \frac{\pi}{4}\right) - 6$

a) amplitude: 4

b) period: π

c) horizontal shift: $-\frac{\pi}{8}$

d) phase shift: $-\frac{\pi}{4}$

b) $T = \frac{2\pi}{2\omega} = \frac{2\pi}{2} = \pi$

c) $\frac{\frac{\pi}{4}}{2} = \frac{\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{8}$

12. For the graph in *Figure 1*;

a) Estimate the period, amplitude, and vertical shift.

Period: 2 Amplitude: 2 Vertical Shift: 1

b) Find a possible sinusoidal formula.

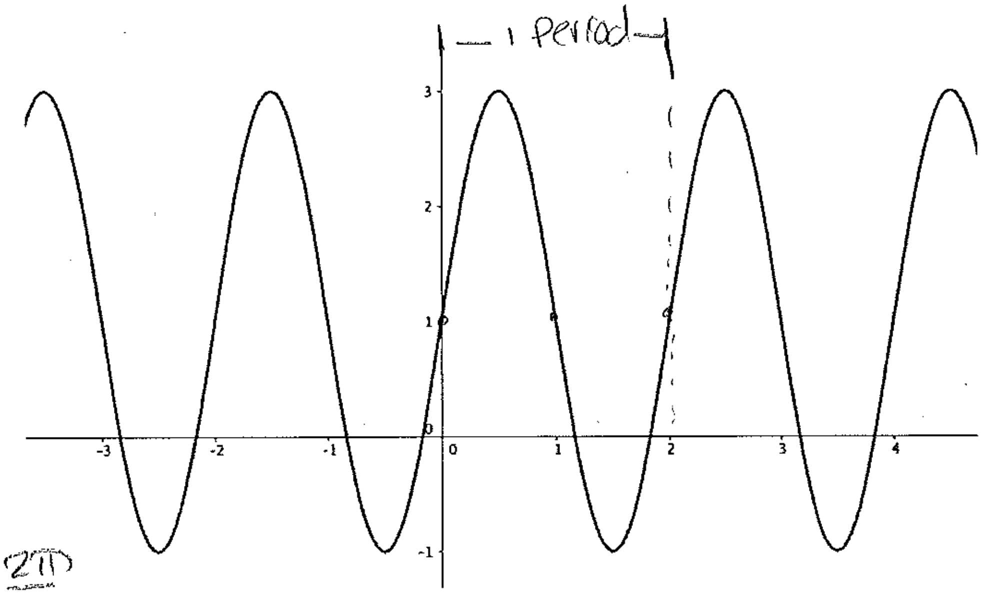


Figure 1

$T = \frac{2\pi}{w}$

$= \frac{2\pi}{\pi} = 2$

Formula: $y = 2\sin(\pi x) + 1$

13. Graph $y = 3\cos\left(\pi x + \frac{\pi}{2}\right) - 1$. Show all steps and work leading up to the graph. Graph 2 cycles

$y = 3\cos\left[\pi\left(x + \frac{1}{2}\right)\right] - 1$

$T = 2 \Rightarrow 2 \div 4 = \frac{1}{2}$

$A = 3$

$w = \pi$

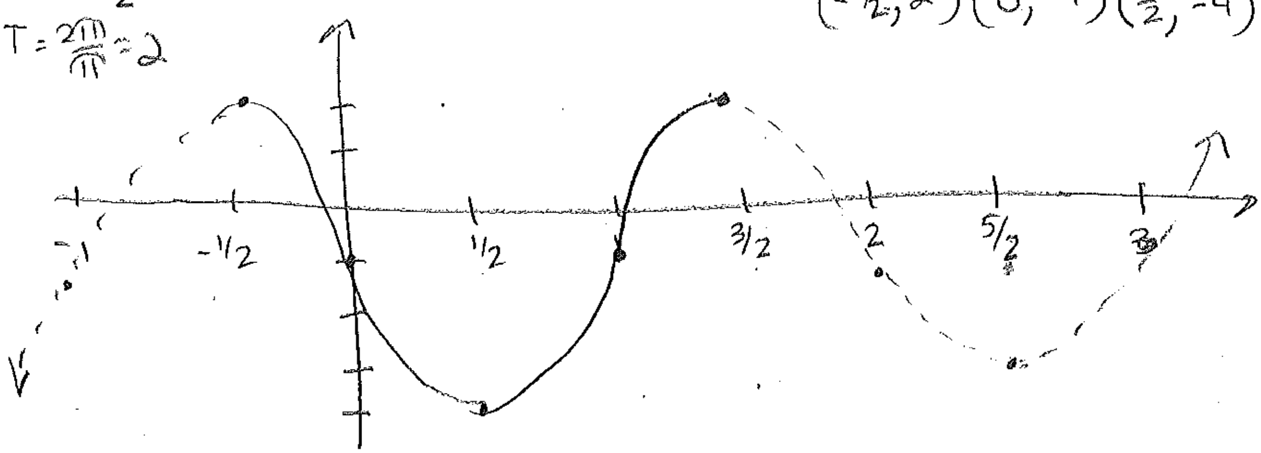
$V.S = -1$

$H.S = -\frac{1}{2}$

$T = \frac{2\pi}{\pi} = 2$

$-\frac{1}{2}$ 0 $\frac{1}{2}$ 1 $\frac{3}{2}$

$(-\frac{1}{2}, 2)\ (0, -1)\ (\frac{1}{2}, -4)\ (1, -1),\ (\frac{3}{2}, 2)$



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PRACTICE MIDTERM I

CALCULATOR

1. Convert 53.72° to $D° M'S"$. Show all work.

$= 53° + .72°$

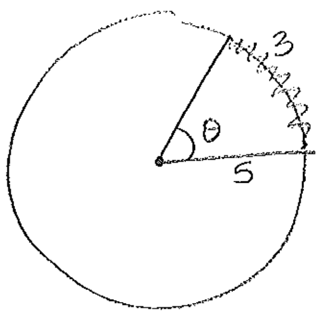
$= 53° + .72(60')$

$= 53° + 43' + .2(60")$

$= 53° + 43' + 12"$

$= 53°43'12"$

2. You walk 3 miles around a circular lake. Given an angle in exact radians and approximate degrees, which represent your final position relative to your starting point if the radius of the lake is 5 miles. Show all your work.



$s = \theta r$

$\frac{3}{5} = \frac{\theta \cancel{5}}{\cancel{5}}$

$\theta = 3/5$

$\theta = .6$

$.6 \cdot \frac{180°}{\pi} = 34.38°$

3. Find the area of the sector of a circle of radius 7 ft formed by an angle of 35°.

$A = \frac{1}{2} r^2 \theta$

$35° \cdot \frac{\pi}{180°} = \frac{35\pi}{180} \approx .6109$

$A = \frac{1}{2}(7)^2\left(\frac{35\pi}{80}\right)$

$\approx 14.97 \text{ ft}^2$