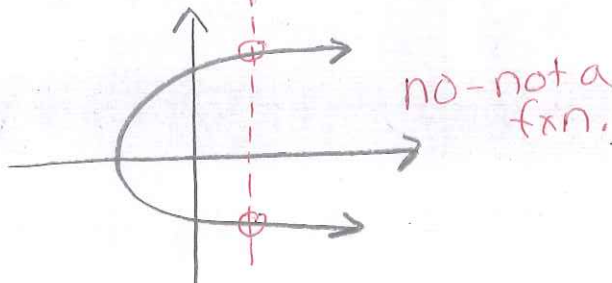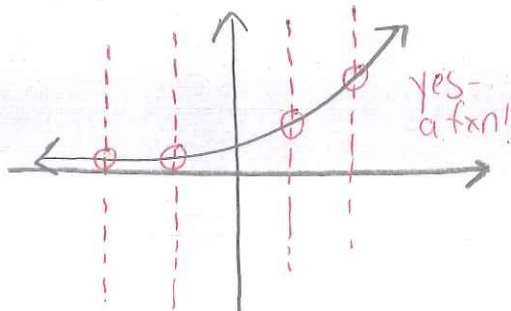## 1.2 The Graph of a Function

From a graph, we can determine if the graph is a function by using the _vertical line test_.
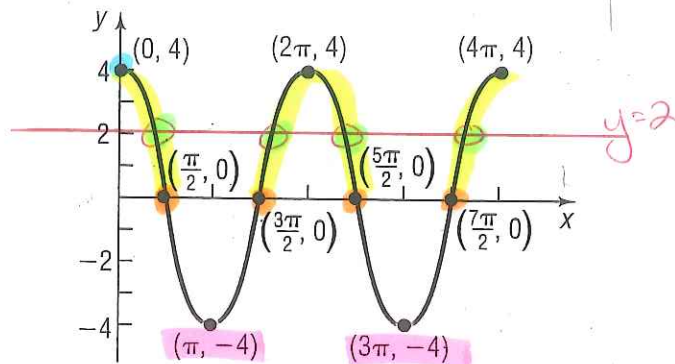
 yes- a fxn!

 no-not a fxn.

Information can be extracted from a given graph, such as _intercepts, domain range, "y" for any "x" in the domain, "x" for any "y" in the range, etc._

Example 1. Refer to the graph.

a) What are $f(0)$, $f\left(\frac{3\pi}{2}\right)$, and $f(3\pi)$?

$f(0) = 4$ $f(3\pi) = -4$
$f\left(\frac{3\pi}{2}\right) = 0$

y
(0, 4)   (2π, 4)   (4π, 4)
4
$y = 2$
2
$\left(\frac{\pi}{2}, 0\right)$ $\left(\frac{5\pi}{2}, 0\right)$
$\left(\frac{3\pi}{2}, 0\right)$ $\left(\frac{7\pi}{2}, 0\right)$ x
-2
-4
(π, -4)   (3π, -4)

b) What is the domain of f? What is the range of f?

D: $[0, 4\pi]$

R: $-4 \leq y \leq 4$ OR $[-4, 4]$

c) List the intercepts.

x int $\left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right), \left(\frac{5\pi}{2}, 0\right), \left(\frac{7\pi}{2}, 0\right)$

y int: $(0, 4)$

d) How many times does the line y = 2 intersect the graph?

4 times

e) For what values of x does $f(x) = -4$?

when y = -4, what values of x are there?

π, 3π

f) For what values of x is $f(x) > 0$?

when y is positive

3 intervals:

$\left[0, \frac{\pi}{2}\right], \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right], \left[\frac{7\pi}{2}, 4\pi\right]$

Information can be extracted from a given function.

Example 2. Consider the function: $f(x) = \frac{x+1}{x+2}$

a) Find the domain of f.

$$x + 2 \neq 0$$
$$-2 \quad -2$$
$$x \neq -2$$

$$D: x \neq -2$$
$$\text{OR}$$
$$\{x \mid x \neq -2\}$$
$$\text{OR}$$
$$(-\infty, -2), (-2, \infty)$$

b) Is the point $(1, \frac{1}{2})$ on the graph of f?

$x = 1, \ y = \frac{1}{2} \Rightarrow$ substitute into $f(x)$

$$\frac{1}{2} \overset{?}{=} \frac{1+1}{1+2}$$

$$\frac{1}{2} \neq \frac{2}{3}$$

FALSE

No, $(1, \frac{1}{2})$ is not on the graph.

c) If $x = 2$, what is $f(x)$? What point is on the graph of f?

$$f(2) = \frac{2+1}{2+2}$$
$$= \frac{3}{4}$$

$f(2) = \frac{3}{4} \Rightarrow (2, \frac{3}{4})$ is on the graph

d) If $f(x) = 2$, what is x? What point is on the graph of f?

$\hookrightarrow y = 2$, solve for "x"

$$\frac{2}{1} = \frac{x+1}{x+2}$$

$$2(x+2) = x+1$$

$$2x + 4 = x + 1$$

$$2x + 4 = x + 1$$
$$\quad -x \quad -x$$
$$x + 4 = 1$$
$$\quad -4 \quad -4$$
$$x = -3 \Rightarrow (-3, 2) \text{ is a point on } f.$$

e) What are the x-intercepts of the graph of f (if any)? What point(s) are on the graph of f?

x-int: Set $y = 0$
y-int: Set $x = 0$

$$f(0) = \frac{0+1}{0+2} = \frac{1}{2}$$

$$\boxed{(0, \frac{1}{2})}$$

$$0 = \frac{x+1}{x+2}$$

$$\frac{0}{1} = \frac{x+1}{x+2}$$

$$x + 1 = 0$$

$$x + 1 = 0$$
$$\quad -1 \quad -1$$
$$x = -1$$

$$\boxed{(-1, 0)}$$