Pre Calc
NOTES

Th 1/26/17

3.5 The Graph of Rational Function

END BEHAVIOR

Consider two scenarios when determining the end behavior of a rational function. (recall n is the degree of numerator, and m is degree of denominator).

a. $n<m$ *(HA is* $y = 0$*)*; as $x \to \infty, f(x) = 0$ & $x \to -\infty, f(x) = 0$ *or* $\lim_{x\to\pm\infty} f(x) = 0$.
b. $n=m$; recall the *HA* is the ratio of the coefficients. So, as $x \to \infty, f(x) = HA$ & $x \to -\infty, f(x) = HA$ *or* $\lim_{x\to\pm\infty} f(x) = HA$.

ANALYZING THE GRAPH OF A RATIONAL FUNCTION R

STEP 1: Factor the numerator and denominator of R. Find the domain of the rational function.

STEP 2: Write R in lowest terms.

STEP 3: Locate the intercepts of the graphs. Determine the behavior of the graph of R at each x-intercept.

STEP 4: Determine the vertical asymptotes. Graph each vertical asymptote using a dashed line.

STEP 5: Determine the horizontal asymptote, if one exists. Determine points, if any, at which the graph of R intersects this asymptote. Graph the asymptote using a dashed line. Plot any points at which the graph of R intersects the asymptote.

STEP 6: Use the zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x=axis by choosing a number (x value) in each interval and evaluating R there. Plot the points found.

STEP 7: Use the results obtained in Steps 1 through 6 to graph R.

Example: Analyze & Graph $R(x) = \frac{x-1}{x^2-4}$

STEP 1: Domain
STEP 2: Lowest terms
STEP 3: Intercepts
STEP 4: VA
STEP 5: HA
[graph intercepts & asymptotes]
STEP 6: Intervals
STEP 7: Graph

Example: Analyze & Graph $R(x) = \frac{x-1}{x^2-4}$

Step 1: $R(x) = \frac{x-1}{x^2-4} = \frac{x-1}{(x+2)(x-2)}$ D: $\{x | x \neq 2, -2\}$

Step 2: already in lowest terms

Step 3: x int (when y=0)

$0 = \frac{x-1}{x^2-4} \Rightarrow x-1=0$

$+1 \quad +1$

$x = 1$

(1,0)

y int (when x=0)

$R(x) = \frac{0-1}{0^2-4} = \frac{-1}{-4} = \frac{1}{4}$

$(0, \frac{1}{4})$

Behavior

zero	m	T/C
1	1	C

Step 4: VA @ x=2, x=-2

Step 5: HA @ y=0 (b/c n<m)

(Graph intercepts + asymptotes)

Step 6:

$(-\infty, -2)$	$(-2, 1)$	$(1, 2)$	$(2, \infty)$
−	+	−	+

Step 7: