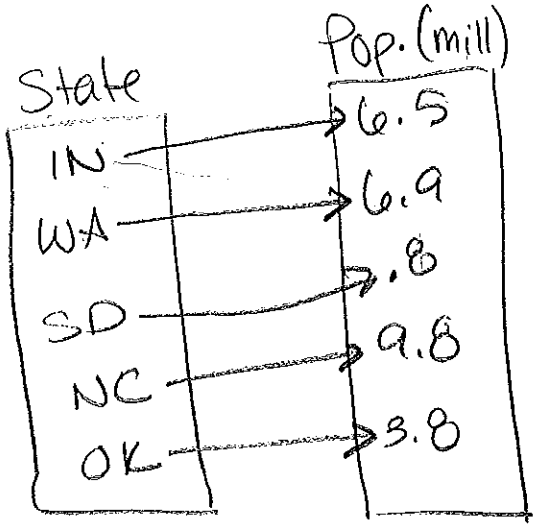
PreCalc Assign: 11-25, 28, 33, 34, 41, 43-48 M 10/31/16

4.2: One-to-One & Inverse Fxns

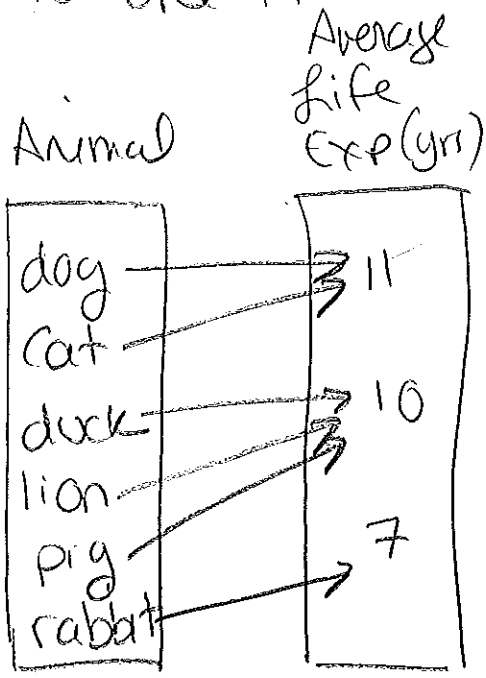
A fxn is one-to-one if any two different inputs in the domain correspond to two different outputs in the range. That is if x_1 and x_2 are any two different inputs of a fxn, f, then f is one-to-one if

$$f(x_1) \neq f(x_2)$$

ex



One-to-one



Not one-to-one

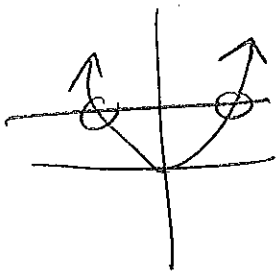
ex $\{(-2,6),(-1,3),(0,2),(1,5),(2,8)\}$ yes

is this a one-to-one fxn?

Horizontal Line Test: If every horiz. line intersects the graph of a function in at most one point, then the function is one-to-one.

a) Is $y = x^2$ a one-to-one?

No

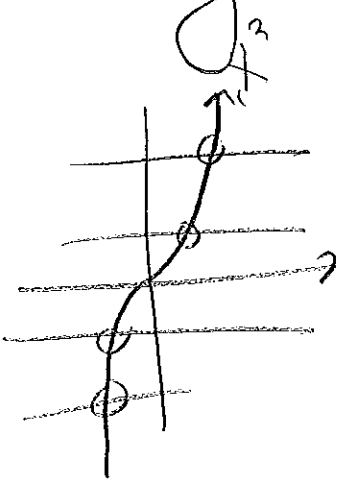


Inv. fxn: $y = x^2$

$\sqrt{x} = \sqrt{y^2}$

$y = \pm\sqrt{x}$ not fxn

b) Is $y = x^3$ a one-to-one? Yes



Inv-fxn: $y = x^3$

$x = y^3$

$\sqrt[3]{x} = \sqrt[3]{y^3}$

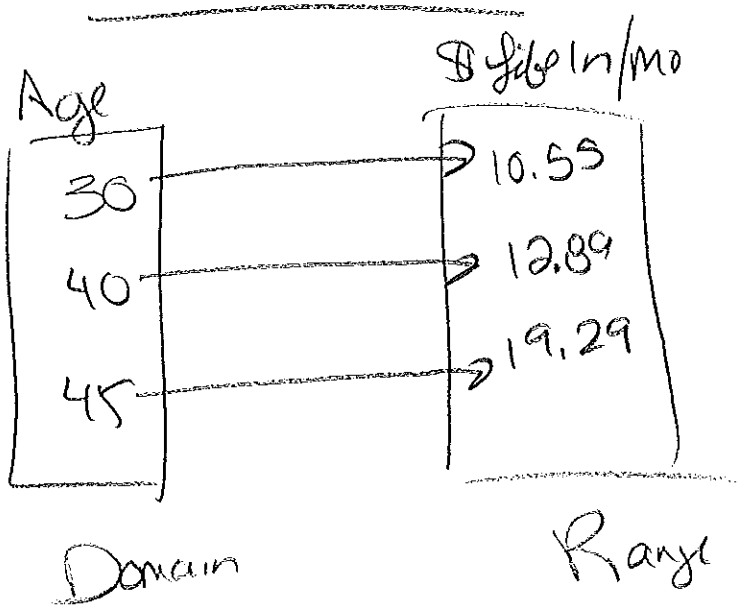
$y = \sqrt[3]{x}$ is a fxn

∴ one-to-one

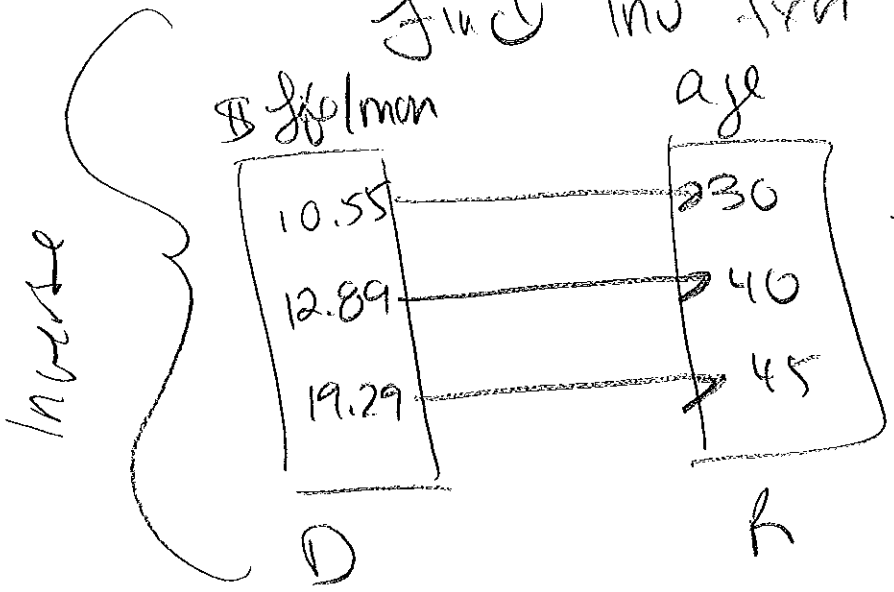
* If a fxn is either increasing or decreasing (but not both), then it is a one-to-one fxn.

Inv. fxn

(27)



find inv fxn



this is a fxn
∴ it is a one-to-one

Verifying Inverses

Given two fxns, $f(x)$ and $g(x)$, verify that they are inverse

① $f(g(x)) = x$

② $g(f(x)) = x$

Notation: If $f(x)$ is a function then its inverse is $f^{-1}(x)$

(35) $f(x) = 4x - 8$, $g(x) = \frac{x}{4} + 2$

a) $f(g(x)) = 4\left(\frac{x}{4} + 2\right) - 8$

$= 4\left(\frac{x}{4}\right) + (4)(2) - 8$

$= x + 8 - 8$

$= x$ ✓

b) $g(f(x)) = \frac{(4x - 8)}{4} + 2$

$= \frac{4x}{4} - \frac{8}{4} + 2$

$= x - 2 + 2$

$= x$ ✓

ex) Verify $f(x) = \frac{1}{x-1}$ and $f^{-1}(x) = \frac{1}{x} + 1$ are inverses.

① $f(f^{-1}(x)) = \dfrac{1}{\left(\frac{1}{x}+1\right)-1}$

$= \dfrac{1}{\frac{1}{x}}$

$= 1 \cdot \frac{x}{1}$

$= x$ ✓

② $f^{-1}(f(x)) = \dfrac{1}{\left(\frac{1}{x-1}\right)} + 1$

$= \dfrac{x-1}{1} + 1$

$= x - 1 + 1$

$= x$ ✓

Graph of the Inverse

the inverse is a reflection across the line $y = x$