

4.5 Properties of Logs

$$\log_a(1) = 0$$

Proof: $\log_a 1 = 0$
 $a^0 = 1$
 $1 = 1 \checkmark$

$$\log_a a = 1$$

Proof: $\log_a a = 1$
 $a^1 = a$
 $a = a \checkmark$

Properties

$$\textcircled{1} a^{\log_a m} = m$$

ex) $2^{\log_2 5} = 5$

$$\textcircled{2} \log_a a^r = r$$

ex) $\log_3 3^x = x$

$$\textcircled{3} \log_a(MN) = \log_a M + \log_a N$$

"condensed"
"expanded"

ex) $\log_4(2x) = \log_4(2) + \log_4(x)$

$$\textcircled{4} \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\textcircled{5} \log_a M^r = r \log_a M$$

"power rule"

ex) $\ln x^2 = 2 \ln(x)$

$$\textcircled{6} e^{\ln a^x} = a^x \quad / \quad \ln(e^x) = x$$

Write $\log_a(x\sqrt{x^2+1})$ as sum

$$\log_a(x) + \log_a(\sqrt{x^2+1})$$

Write $\ln\left[\frac{x^2}{(x-1)^3}\right]$ as a difference

$$\ln(x^2) - \ln[(x-1)^3] \quad \text{rule \#4}$$

$$2\ln(x) - 3\ln(x-1) \quad \text{rule \#5}$$

Write $\log_a\left[\frac{\sqrt{x^2+1}}{x^3(x+1)^4}\right]$, $x > 0$, as a sum + difference

$$\log_a(\sqrt{x^2+1}) - \log_a[x^3(x+1)^4] \quad \text{rule \#4}$$

$$\sqrt{x} = x^{1/2} \quad \log_a(x^2+1)^{1/2} - [\log_a x^3 + \log_a(x+1)^4] \quad \text{rule \#3}$$

$$\frac{1}{2}\log_a(x^2+1) - 3\log_a x + 4\log_a(x+1) \quad \text{rule \#5}$$

More properties

⑦ If $M=N$, then $\log_a M = \log_a N$

⑧ If $\log_a M = \log_a N$, then $M=N$

⑨ Change of Base formula

$$\log_a M = \frac{\log M}{\log a} \quad / \quad \log_a M = \frac{\ln M}{\ln a}$$

ex) Approximate $\log_5 89$

$$\log_5 89 = \frac{\ln 89}{\ln 5}$$

} change of
base
formula

$$\approx 2.79$$



4.5 assignment: 1-12, 13-25, 33-40,
47, 49, 51, 57, 63, 71-78, 101