4.7 Financial models

A. Simple Interest : $I = Prt$

where I = interest in dollars
P = principal / initial amount in dollars
t = # years
r = interest rate (in decimal form)

ex) a bank pays interest of 2% per annum paid annually. How much interest would be earned after 1 year if $1,000 is deposited.

$$I = 1{,}000(.02)(1)$$
$$= \boxed{\$20}$$

B. Compounded Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A = amount of $ after t years
P = principal / initial $
r = interest rate
t = # years
n = # times per year

annually : $n = 1$
semiannually : $n = 2$
quarterly : $n = 4$
monthly : $n = 12$
daily : $n = 365$
weekly : $n = 52$

ex) Investing $1,000 at an annual rate of 10% compounded

a) annually ($n=1$)
b) semiannually ($n=2$)
c) quarterly ($n=4$)
d) monthly ($n=12$)
e) daily ($n=365$)

find the dollar amount after 1 year. Use $A = P\left(1+\frac{r}{n}\right)^{nt}$

a) $A = \$1,000\left(1+\frac{.10}{1}\right)^{1(1)}$
$= \$1,1000$

b) $A = \$1,000\left(1+\frac{.1}{2}\right)^{2(1)}$
$= \$1,102.50$

c) $A = \$1,000\left(1+\frac{.1}{4}\right)^{4(1)}$
$= \$1,103.81$

d) $A = \$1,000\left(1+\frac{.10}{12}\right)^{12(1)}$
$= \$1,104.71$

e) $A = \$1,000\left(1+\frac{.1}{365}\right)^{365(1)}$
$= \$1,105.16$

Notice as n increases, A increases

C. Continuous Compounding : $A = Pe^{rt}$

Keyword: continuous

P = principal
$e \approx 2.71$
r = interest rate
t = # years

ex) Investing $1,000 at an annual rate of 10% compounded continuously for 1 year.

$A = Pe^{rt}$
$= (\$1,000)e^{(.1)(1)}$
$= \$1,105.17$

D. Effective Rate of Interest: the equivalent annual simple interest rate that would yield the same amount as compounding n times per year, or continuously, after 1 year.

Compounding n times per year: $r_e = \left(1 + \frac{r}{n}\right)^n - 1$

Continuous compounding: $r_e = e^r - 1$

The "better deal" is the effective rate which is higher when comparing different options.

Assignment 4.7: 7–13 odd, 23, 25, 27, 31, 33, 35, 39, 41, 43, 50, 53