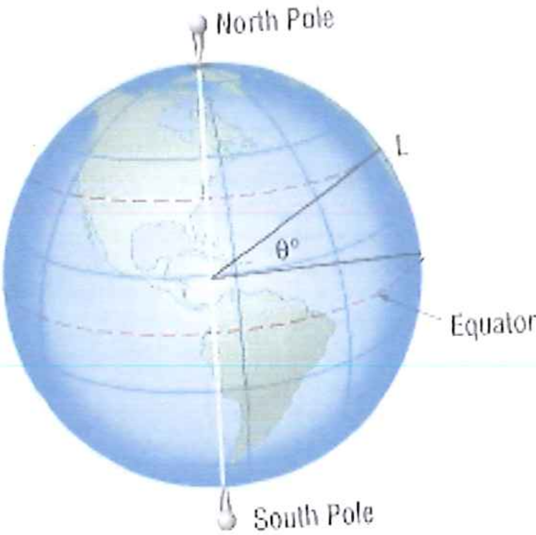
PreCalc

5.1 Angles & Their Measures Part II

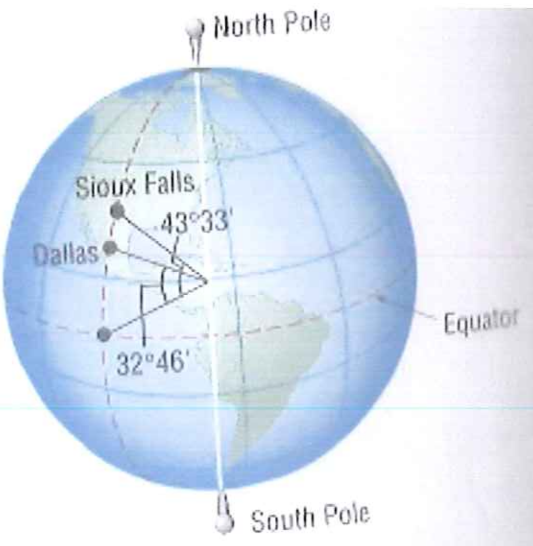
FINDING THE DISTANCE BETWEEN TWO CITIES

horiz

The latitude of a location L is the measure of the angle formed by a ray drawn from the center of the Earth to the equator and a ray drawn from the center of the Earth to L. See figure (a) below. Sioux Falls, SD is due north of Dallas, TX. Find the distance between Sioux Falls (43° 33' north latitude) and Dallas (32° 46' north latitude). See figure (b) below. Assume that the radius of the Earth is 3960 miles.



(a)



(b)

① measure of θ btwn two cities is $43°33' - 32°46'$.

$43°\ 33\left(\frac{1}{60}\right) \quad 32°.46\left(\frac{1}{60}\right)$

$= 43.55° \quad = 32.77°$

$43.55° - 32.77° = 10.78°$

$10° + .78(60') = 10° + 46.8'$

$= \boxed{10°\ 47'}$

② find S. between two cities

$S = r\theta \qquad \theta = 10°47' \approx 10.7833° = 10.7833° \cdot \frac{\pi^c}{180°} \approx 0.1882^c$

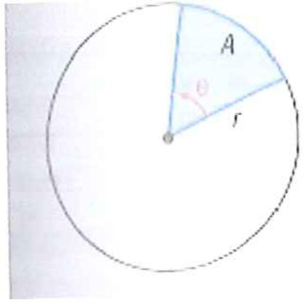
$\left[47' = 47\left(\frac{1}{60}\right)°\right]$

$S = 3690 \cdot 0.1882^c \approx 745$ miles

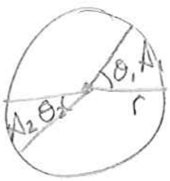
AREA OF A SECTOR OF A CIRCLE

The area A of the sector of a circle of radius r formed by a central angle of θ radians is given by...

$$\boxed{A = \frac{1}{2}r^2\theta}$$



Derivation



$$\frac{\theta_1}{\theta_2} = \frac{A_1}{A_2}$$

$$A_1 = A_2\frac{\theta_1}{\theta_2} = \pi r^2\left(\frac{\theta_1}{\theta_2}\right) = \cancel{\pi} r^2\left(\frac{\theta_1}{2\cancel{\pi}}\right) = \frac{1}{2}r^2\theta_1$$

$$\left[\begin{array}{l} A = \pi r^2 \\ \theta = 2\pi \end{array}\right]$$

Example. Find the area of the sector of a circle of radius 2 ft formed by an angle of 30°. Round to two decimal places.

$r = 2\text{ ft},\ \theta = 30^\circ$ $\qquad 30^\circ \cdot \frac{\pi}{180^\circ} = \frac{30\pi}{180} = \frac{\pi}{6}$

$$A = \frac{1}{2}(2)^2\left(\frac{\pi}{6}\right) = \frac{1}{2}(4)\left(\frac{\pi}{6}\right) = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3} \approx 1.05\text{ ft}^2$$

LINEAR SPEED OF AN OBJECT TRAVELING IN CIRCULAR MOTION

Suppose that an object moves around a circle of radius r at a constant speed. If s is the distance traveled in time t around this circle, then the **linear speed** v of the object is defined as

$$\boxed{v = \frac{s}{t}}$$

The **angular speed** ω (Greek letter omega) of this object is the angle θ swept out (measured in radians), divided by the elapsed time t.

$$\boxed{\omega = \frac{\theta}{t}}$$

Angular speed is the way the turning engine is described.

Example. An engine idling at 900rpm (revolutions per minute) is one that rotates at an angular speed of...

angular speed (ω) is in radians

$$900 \text{ rpm} = 900 \frac{\text{rev}}{\text{min}} \cdot \underbrace{2\pi \frac{\text{radians}}{\text{rev}}}_{\text{conversion factor}} = 1800\pi \frac{\text{radians}}{\text{min}}$$

The relationship between linear speed and angular speed is given by

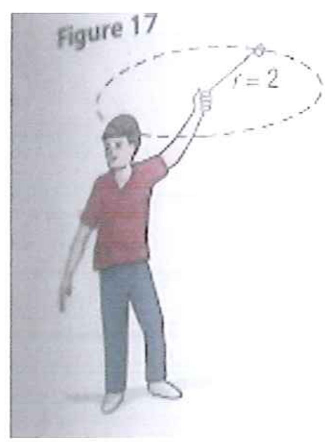
$$\boxed{v = r\omega}$$

...where ω is measured in radians per unit time.

Linear speed has dimensions of length per unit of time. Angular speed has the dimensions of radians per unit of time. If angular speed is given in terms of *revolutions* per unit of time, convert to *radians* per unit of time, using the fact that 1 revolution = 2π radians.

Example. A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute (rpm). Find the linear speed of the rock when it is released.

$r = 2 \text{ ft}$ $\quad \omega = 180 \frac{\text{rev}}{\text{min}}$



$$180 \frac{\text{rev}}{\text{min}} \cdot 2\pi \frac{\text{radians}}{\text{rev}} = 360\pi \, ^c/\text{min}$$

$$v = r\omega = 2 \text{ ft} \cdot 360\pi^c/\text{min}$$
$$= 720\pi \text{ ft/min}$$
$$\approx 2262 \text{ ft/min}$$

To convert to miles/hr: 5280 ft = 1 mile
1 hr = 60 min

$$\frac{2262 \text{ ft}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \approx 25.7 \text{ miles/hr.}$$

ASSIGN: 91, 98, 99, 100, 101, 107, 108, 119