PreCalc

Name____________________________

Date________________________ Pd 7

PRE-CALCULUS PRACTICE TEST II

No Calculator

1. Fill in the table.

x	$\sin^{-1}(x)$ $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$\cos^{-1}(x)$ $[0,\pi]$	$\tan^{-1}(x)$ $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
-1	$-\frac{\pi}{2}$	π	$-\frac{\pi}{4}$
$\frac{1}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	
$-\frac{1}{2}$	$-\frac{\pi}{6}$	$\frac{2\pi}{3}$	
$-\frac{\sqrt{2}}{2}$	$-\frac{\pi}{4}$	$\frac{3\pi}{4}$	
1	$\frac{\pi}{2}$	0	$\frac{\pi}{4}$
$-\sqrt{3}$			$-\frac{\pi}{3}$

2. Evaluate the expression algebraically.

a. $\tan\left(\cos^{-1}\left(-\frac{1}{5}\right)\right)$

Let $\theta = \cos^{-1}\left(-\frac{1}{5}\right)$

$\cos\theta = -\frac{1}{5} = \frac{x}{r}$

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (5)^2$

$y^2 = 24$

$y = \sqrt{24} = 2\sqrt{6}$

$\tan\theta = \frac{y}{x} = \frac{2\sqrt{6}}{-1} = -2\sqrt{6}$

b. $\cos\left(\tan^{-1}\left(-\frac{1}{5}\right)\right)$

Let $\theta = \tan^{-1}\left(-\frac{1}{5}\right)$

$\tan\theta = -\frac{1}{5} = \frac{y}{x} \begin{matrix}\to(-)\\ \to(+)\end{matrix}$

$5^2 + (-1)^2 = r^2$

$r^2 = 26$

$r = \sqrt{26}$

$\cos\theta = \frac{x}{r} = \frac{5}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}}$

$= \frac{5\sqrt{26}}{26}$

3. Solve the equations algebraically.

a. $\sec(\theta) = 2$. Find all solutions.

$\frac{1}{\cos\theta} = \frac{2}{1}$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2\pi k$

$\theta = \frac{5\pi}{3} + 2\pi k$

d. $2\cos^2\theta + \cos\theta = 0$ over $0 \le \theta < 2\pi$.

$\cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0$ $\quad 2\cos\theta + 1 = 0$

$\theta = 0, \pi$ $\quad \cos\theta = -1/2$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

b. $2\sin^2(\theta) + 1 = 3$ over $-2\pi \le \theta \le 2\pi$.

$2\sin^2\theta = 2$

$\sin^2\theta = 1$

$\sin\theta = 1$ $\quad \sin\theta = -1$

$\theta = \frac{\pi}{2}$ $\quad \theta = \frac{3\pi}{2}$

$\left\{\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}\right\}$

e. $\sin^2\theta = 2\cos\theta + 2$ over $0 \le \theta < 2\pi$.

$1 - \cos^2\theta = 2\cos\theta + 2$

$\cos^2\theta + 2\cos\theta + 1 = 0$

$(\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -1$

$\theta = \pi$

c. $\sin(\theta) = \frac{1}{2}$. Find all solutions.

$\frac{\pi}{6} + 2\pi k$

$\frac{5\pi}{6} + 2\pi k$

f. $\sin(2\theta) - 1 = 0$. Find all solutions.

$\sin(2\theta) = 1$

$\theta = \frac{\pi}{2} + 2\pi k$

$\frac{2\theta = \frac{\pi}{2} + 2\pi k}{2}$

$\theta = \frac{\pi}{4} + \pi k$

4. If $\cos(\alpha) = \frac{\sqrt{5}}{5}$ *where* $0 \le \alpha \le \frac{\pi}{2}$, *and* $\sin(\beta) = -\frac{4}{5}$ *where* $-\frac{\pi}{2} \le \beta < 0$; then find

a. $\sin(\alpha + \beta)$ b. $\cos(\alpha + \beta)$ c. $\sin(\alpha - \beta)$ d. $\cot(\beta)$

$\sin\alpha = \frac{2\sqrt{5}}{5}$

$\cos\beta = \frac{3}{5}$

a) $\sin(\alpha + \beta) = \frac{2\sqrt{5}}{5}$

b) $\cos(\alpha + \beta) = \frac{11\sqrt{5}}{5}$

c) $\sin(\alpha - \beta) = \frac{10\sqrt{5}}{5}$

d) $\cot(\beta) = -3/4$

$\cos^2(\alpha) + \sin^2(\alpha) = 1$

$\left(\frac{\sqrt{5}}{5}\right)^2 + \sin^2(\alpha) = 1$

$\frac{5}{25} + \sin^2(\alpha) = \frac{25}{25}$

$\sin^2(\alpha) = \frac{20}{25}$

$\sin\alpha = \frac{\sqrt{20}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}$

$\sin^2\beta + \cos^2\beta = 1$

$\left(\frac{-4}{5}\right)^2 + \cos^2\beta = 1$

$\frac{16}{25} + \cos^2\beta = \frac{25}{25}$

$\cos^2\beta = \frac{9}{25}$

$\Rightarrow \cos\beta = \frac{3}{5}$

5. Prove the identities.

a. $\sin(x) \cdot \csc(x) - cos^2(x) = sin^2(x)$

$\frac{\cancel{\sin x}}{1} \cdot \frac{1}{\cancel{\sin x}} - \cos^2 x$

$1 - \cos^2 x$

$\sin^2 x$

c. $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin\alpha \, cos\beta$

$\sin\alpha\cos\beta + \sin\beta\cos\alpha + \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$\sin\alpha\cos\beta + \sin\alpha\cos\beta$

$2\sin\alpha\cos\beta$

b. $(1 - cos^2\theta)(1 + cot^2\theta) = 1$

$(\sin^2\theta)(\csc^2\theta)$

$\cancel{\sin^2\theta} \cdot \frac{1}{\cancel{\sin^2\theta}}$

1

e. $tan^2(x)cos^2(x) + cot^2(x)sin^2(x) = 1$

$\frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x} + \frac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \cancel{\sin^2 x}$

$\sin^2 x + \cos^2 x$

1

6. Find the exact values of

a. $\sin(75)$

$\sin(45 + 30)$

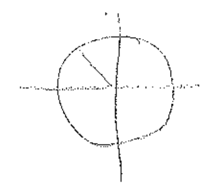
$= \sin 45 \cos 30 + \sin 30 \cos 45$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} + \sqrt{6}}{4}$

b. $\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$



$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

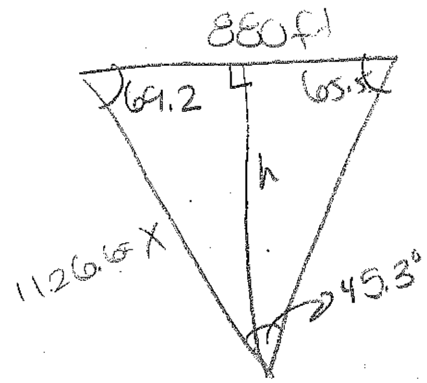
$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{-\sqrt{6} - \sqrt{2}}{4}$

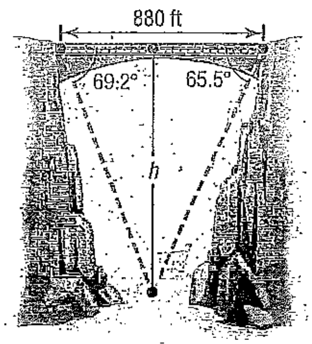
Calculator

1. The highest bridge in the world is the bridge over the Royal Gorge of the Arkansas River in Colorado. Sightings to the same point at water level directly under the bridge are taken from each side of the 880-foot long bridge, as indicated in the figure. How high is the bridge?



$\sin 69.2 = \frac{h}{1126.6}$

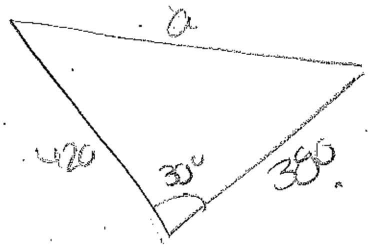
$\boxed{h = 1053.17 \text{ ft}}$



$\frac{\sin 45.3}{880} = \frac{\sin 65.5}{X}$

$X = 1126.6$

2. A boy is flying two kites at the same time. He has 380 feet of line out to one kite and 420 feet of line out to the other kite. He estimates the angle between the two lines to be 30°. Approximate to the nearest tenth (one decimal place) of a foot, the distance between the kites. Show all steps.



$a^2 = (420)^2 + (380)^2 - 2(420)(380)\cos 30$

$a^2 = 44364.7$

$\boxed{a \approx 216.6 \text{ ft}}$