# SUMMARY

| | |
|---|---|
| **Function** | A relation between two sets of real numbers so that each number $x$ in the first set, the domain, has corresponding to it exactly one number $y$ in the second set.<br>A set of ordered pairs $(x, y)$ or $(x, f(x))$ in which no first element is paired with two different second elements.<br>The range is the set of $y$-values of the function that are the images of the $x$-values in the domain.<br>A function $f$ may be defined implicitly by an equation involving $x$ and $y$ or explicitly by writing $y = f(x)$. |
| **Unspecified domain** | If a function $f$ is defined by an equation and no domain is specified, then the domain will be taken to be the largest set of real numbers for which the equation defines a real number. |
| **Function notation** | $y = f(x)$<br>$f$ is a symbol for the function.<br>$x$ is the independent variable or argument.<br>$y$ is the dependent variable.<br>$f(x)$ is the value of the function at $x$, or the image of $x$. |

## 1.1 Assess Your Understanding

**'Are You Prepared?'** *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. The inequality $-1 < x < 3$ can be written in interval notation as ________. (pp. A81–A82)
2. If $x = -2$, the value of the expression $3x^2 - 5x + \frac{1}{x}$ is ______. (pp. A6–A7)
3. The domain of the variable in the expression $\frac{x-3}{x+4}$ is ________. (p. A7)
4. Solve the inequality $3 - 2x > 5$. Graph the solution set. (pp. A84–A86)

### Concepts and Vocabulary

5. If $f$ is a function defined by the equation $y = f(x)$, then $x$ is called the ________ variable and $y$ is the ________ variable.
6. The set of all images of the elements in the domain of a function is called the ______.
7. If the domain of $f$ is all real numbers in the interval $[0, 7]$ and the domain of $g$ is all real numbers in the interval $[-2, 5]$, then the domain of $f + g$ is all real numbers in the interval ____.
8. The domain of $\frac{f}{g}$ consists of numbers $x$ for which $g(x)$ ____ 0 that are in the domains of both ____ and ____.
9. If $f(x) = x + 1$ and $g(x) = x^3$, then ________ $= x^3 - (x + 1)$.
10. *True or False* Every relation is a function.
11. *True or False* The domain of $(f \cdot g)(x)$ consists of the numbers $x$ that are in the domains of both $f$ and $g$.
12. *True or False* The independent variable is sometimes referred to as the argument of the function.
13. *True or False* If no domain is specified for a function $f$, then the domain of $f$ is taken to be the set of real numbers.
14. *True or False* The domain of the function $f(x) = \frac{x^2 - 4}{x}$ is $\{x \mid x \neq \pm 2\}$.

### Skill Building

*In Problems 15–26, determine whether each relation represents a function. For each function, state the domain and range.*

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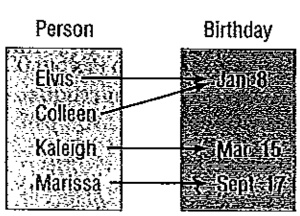


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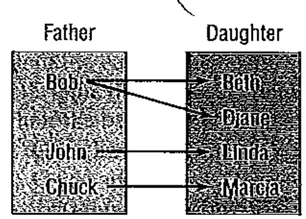

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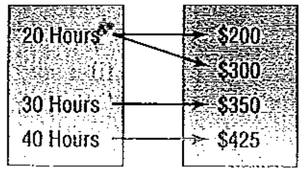


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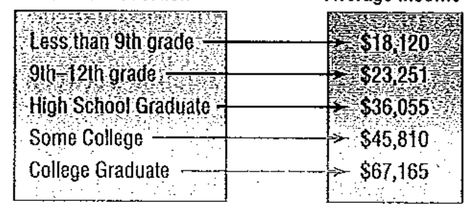



19. $\{(2, 6), (-3, 6), (4, 9), (2, 10)\}$
20. $\{(-2, 5), (-1, 3), (3, 7), (4, 12)\}$
21. $\{(1, 3), (2, 3), (3, 3), (4, 3)\}$
22. $\{(0, -2), (1, 3), (2, 3), (3, 7)\}$
23. $\{(-2, 4), (-2, 6), (0, 3), (3, 7)\}$
24. $\{(-4, 4), (-3, 3), (-2, 2), (-1, 1), (-4, 0)\}$
25. $\{(-2, 4), (-1, 1), (0, 0), (1, 1)\}$
26. $\{(-2, 16), (-1, 4), (0, 3), (1, 4)\}$

*In Problems 27–38, determine whether the equation defines y as a function of x.*

27. $y = x^2$
28. $y = x^3$
29. $y = \dfrac{1}{x}$
30. $y = |x|$
31. $y^2 = 4 - x^2$
32. $y = \pm\sqrt{1 - 2x}$
33. $x = y^2$
34. $x + y^2 = 1$
35. $y = 2x^2 - 3x + 4$
36. $y = \dfrac{3x - 1}{x + 2}$
37. $2x^2 + 3y^2 = 1$
38. $x^2 - 4y^2 = 1$

*In Problems 39–46, find the following for each function:*

(a) $f(0)$ (b) $f(1)$ (c) $f(-1)$ (d) $f(-x)$ (e) $-f(x)$ (f) $f(x + 1)$ (g) $f(2x)$ (h) $f(x + h)$

39. $f(x) = 3x^2 + 2x - 4$
40. $f(x) = -2x^2 + x - 1$
41. $f(x) = \dfrac{x}{x^2 + 1}$
42. $f(x) = \dfrac{x^2 - 1}{x + 4}$
43. $f(x) = |x| + 4$
44. $f(x) = \sqrt{x^2 + x}$
45. $f(x) = \dfrac{2x + 1}{3x - 5}$
46. $f(x) = 1 - \dfrac{1}{(x + 2)^2}$

*In Problems 47–62, find the domain of each function.*

47. $f(x) = -5x + 4$
48. $f(x) = x^2 + 2$
49. $f(x) = \dfrac{x}{x^2 + 1}$
50. $f(x) = \dfrac{x^2}{x^2 + 1}$
51. $g(x) = \dfrac{x}{x^2 - 16}$
52. $h(x) = \dfrac{2x}{x^2 - 4}$
53. $F(x) = \dfrac{x - 2}{x^3 + x}$
54. $G(x) = \dfrac{x + 4}{x^3 - 4x}$
55. $h(x) = \sqrt{3x - 12}$
56. $G(x) = \sqrt{1 - x}$
57. $f(x) = \dfrac{4}{\sqrt{x - 9}}$
58. $f(x) = \dfrac{x}{\sqrt{x - 4}}$
59. $p(x) = \sqrt{\dfrac{2}{x - 1}}$
60. $q(x) = \sqrt{-x - 2}$
61. $P(t) = \dfrac{\sqrt{t - 4}}{3t - 21}$
62. $h(z) = \dfrac{\sqrt{z + 3}}{z - 2}$

*In Problems 63–72, for the given functions f and g, find the following. For parts (a)–(d), also find the domain.*

(a) $(f + g)(x)$ (b) $(f - g)(x)$ (c) $(f \cdot g)(x)$ (d) $\left(\dfrac{f}{g}\right)(x)$

(e) $(f + g)(3)$ (f) $(f - g)(4)$ (g) $(f \cdot g)(2)$ (h) $\left(\dfrac{f}{g}\right)(1)$

63. $f(x) = 3x + 4;\ g(x) = 2x - 3$
64. $f(x) = 2x + 1;\ g(x) = 3x - 2$
65. $f(x) = x - 1;\ g(x) = 2x^2$
66. $f(x) = 2x^2 + 3;\ g(x) = 4x^3 + 1$
67. $f(x) = \sqrt{x};\ g(x) = 3x - 5$
68. $f(x) = |x|;\ g(x) = x$
69. $f(x) = 1 + \dfrac{1}{x};\ g(x) = \dfrac{1}{x}$
70. $f(x) = \sqrt{x - 1};\ g(x) = \sqrt{4 - x}$
71. $f(x) = \dfrac{2x + 3}{3x - 2};\ g(x) = \dfrac{4x}{3x - 2}$
72. $f(x) = \sqrt{x + 1};\ g(x) = \dfrac{2}{x}$

73. Given $f(x) = 3x + 1$ and $(f + g)(x) = 6 - \dfrac{1}{2}x$, find the function $g$.

74. Given $f(x) = \dfrac{1}{x}$ and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x + 1}{x^2 - x}$, find the function $g$.

*In Problems 75–82, find the difference quotient of f; that is, find* $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$, *for each function. Be sure to simplify.*

75. $f(x) = 4x + 3$
76. $f(x) = -3x + 1$
77. $f(x) = x^2 - x + 4$
78. $f(x) = 3x^2 - 2x + 6$
79. $f(x) = \frac{1}{x^2}$
80. $f(x) = \frac{1}{x+3}$
81. $f(x) = \sqrt{x}$ [**Hint:** Rationalize the numerator.]
82. $f(x) = \sqrt{x+1}$

## Applications and Extensions

83. If $f(x) = 2x^3 + Ax^2 + 4x - 5$ and $f(2) = 5$, what is the value of $A$?

84. If $f(x) = 3x^2 - Bx + 4$ and $f(-1) = 12$, what is the value of $B$?

85. If $f(x) = \frac{3x+8}{2x-A}$ and $f(0) = 2$, what is the value of $A$?

86. If $f(x) = \frac{2x-B}{3x+4}$ and $f(2) = \frac{1}{2}$, what is the value of $B$?

87. If $f(x) = \frac{2x-A}{x-3}$ and $f(4) = 0$, what is the value of $A$? Where is $f$ not defined?

88. If $f(x) = \frac{x-B}{x-A}$, $f(2) = 0$, and $f(1)$ is undefined, what are the values of $A$ and $B$?

89. **Geometry** Express the area $A$ of a rectangle as a function of the length $x$ if the length of the rectangle is twice its width.

90. **Geometry** Express the area $A$ of an isosceles right triangle as a function of the length $x$ of one of the two equal sides.

91. **Constructing Functions** Express the gross salary $G$ of a person who earns \$10 per hour as a function of the number $x$ of hours worked.

92. **Constructing Functions** Tiffany, a commissioned sales person, earns \$100 base pay plus \$10 per item sold. Express her gross salary $G$ as a function of the number $x$ of items sold.

93. **Population as a Function of Age** The function

$$P(a) = 0.004a^2 - 3.792a + 317.946$$

represents the population $P$ (in millions) of Americans that were $a$ years of age or older in 2011.

*Source:* U.S. Census Bureau

(a) Identify the dependent and independent variables.
(b) Evaluate $P(20)$. Provide a verbal explanation of the meaning of $P(20)$.
(c) Evaluate $P(0)$. Provide a verbal explanation of the meaning of $P(0)$.

94. **Number of Rooms** The function

$$N(r) = -2.08r^2 + 22.901r - 36.06$$

represents the number $N$ of housing units (in millions) in 2011 that had $r$ rooms, where $r$ is an integer and $2 \le r \le 9$.

*Source:* U.S. Census Bureau

(a) Identify the dependent and independent variables.
(b) Evaluate $N(3)$. Provide a verbal explanation of the meaning of $N(3)$.

95. **Effect of Gravity on Earth** If a rock falls from a height of 20 meters on Earth, the height $H$ (in meters) after $x$ seconds is approximately

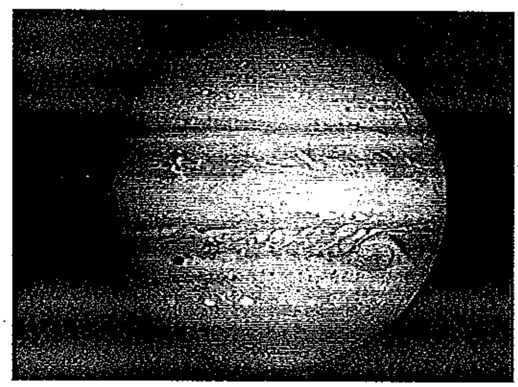
$$H(x) = 20 - 4.9x^2$$

(a) What is the height of the rock when $x = 1$ second? $x = 1.1$ seconds? $x = 1.2$ seconds? $x = 1.3$ seconds?
(b) When is the height of the rock 15 meters? When is it 10 meters? When is it 5 meters?
(c) When does the rock strike the ground?

96. **Effect of Gravity on Jupiter** If a rock falls from a height of 20 meters on the planet Jupiter, its height $H$ (in meters) after $x$ seconds is approximately

$$H(x) = 20 - 13x^2$$

(a) What is the height of the rock when $x = 1$ second? $x = 1.1$ seconds? $x = 1.2$ seconds?
(b) When is the height of the rock 15 meters? When is it 10 meters? When is it 5 meters?
(c) When does the rock strike the ground?



97. **Cost of Trans-Atlantic Travel** A Boeing 747 crosses the Atlantic Ocean (3000 miles) with an airspeed of 500 miles per hour. The cost $C$ (in dollars) per passenger is given by

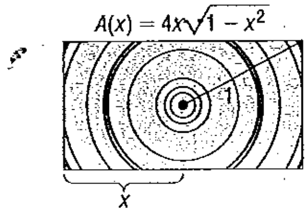
$$C(x) = 100 + \frac{x}{10} + \frac{36{,}000}{x}$$

where $x$ is the ground speed (airspeed $\pm$ wind).

(a) What is the cost per passenger for quiescent (no wind) conditions?
(b) What is the cost per passenger with a head wind of 50 miles per hour?
(c) What is the cost per passenger with a tail wind of 100 miles per hour?
(d) What is the cost per passenger with a head wind of 100 miles per hour?

98. **Cross-sectional Area** The cross-sectional area of a beam cut from a log with radius 1 foot is given by the function $A(x) = 4x\sqrt{1-x^2}$, where $x$ represents the length, in feet, of half the base of the beam. See the figure on the next page. Determine the cross-sectional area of the beam if the length of half the base of the beam is as follows:

(a) One-third of a foot
(b) One-half of a foot
(c) Two-thirds of a foot




99. **Economics** The **participation rate** is the number of people in the labor force divided by the civilian population (excludes military). Let $L(x)$ represent the size of the labor force in year $x$ and $P(x)$ represent the civilian population in year $x$. Determine a function that represents the participation rate $R$ as a function of $x$.

100. **Crimes** Suppose that $V(x)$ represents the number of violent crimes committed in year $x$ and $P(x)$ represents the number of property crimes committed in year $x$. Determine a function $T$ that represents the combined total of violent crimes and property crimes in year $x$.

101. **Health Care** Suppose that $P(x)$ represents the percentage of income spent on health care in year $x$ and $I(x)$ represents income in year $x$. Determine a function $H$ that represents total health care expenditures in year $x$.

102. **Income Tax** Suppose that $I(x)$ represents the income of an individual in year $x$ before taxes and $T(x)$ represents the individual's tax bill in year $x$. Determine a function $N$ that represents the individual's net income (income after taxes) in year $x$.

103. **Profit Function** Suppose that the revenue $R$, in dollars, from selling $x$ cell phones, in hundreds, is $R(x) = -1.2x^2 + 220x$. The cost $C$, in dollars, of selling $x$ cell phones, in hundreds, is $C(x) = 0.05x^3 - 2x^2 + 65x + 500$.
(a) Find the profit function, $P(x) = R(x) - C(x)$.
(b) Find the profit if $x = 15$ hundred cell phones are sold.
(c) Interpret $P(15)$.

104. **Profit Function** Suppose that the revenue $R$, in dollars, from selling $x$ clocks is $R(x) = 30x$. The cost $C$, in dollars, of selling $x$ clocks is $C(x) = 0.1x^2 + 7x + 400$.
(a) Find the profit function, $P(x) = R(x) - C(x)$.
(b) Find the profit if $x = 30$ clocks are sold.
(c) Interpret $P(30)$.

105. **Stopping Distance** When the driver of a vehicle observes an impediment, the total stopping distance involves both the reaction distance (the distance the vehicle travels while the driver moves his or her foot to the brake pedal) and the braking distance (the distance the vehicle travels once the brakes are applied). For a car traveling at a speed of $v$ miles per hour, the reaction distance $R$, in feet, can be estimated by $R(v) = 2.2v$. Suppose that the braking distance $B$, in feet, for a car is given by $B(v) = 0.05v^2 + 0.4v - 15$.
(a) Find the stopping distance function $D(v) = R(v) + B(v)$.
(b) Find the stopping distance if the car is traveling at a speed of 60 mph.
(c) Interpret $D(60)$.

106. Some functions $f$ have the property that $f(a + b) = f(a) + f(b)$ for all real numbers $a$ and $b$. Which of the following functions have this property?
(a) $h(x) = 2x$ (b) $g(x) = x^2$
(c) $F(x) = 5x - 2$ (d) $G(x) = \frac{1}{x}$

## Discussion and Writing

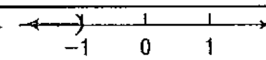
107. Are the functions $f(x) = x - 1$ and $g(x) = \frac{x^2 - 1}{x + 1}$ the same? Explain.

108. Investigate when, historically, the use of the function notation $y = f(x)$ first appeared.

109. Find a function $H$ that multiplies a number $x$ by 3 and then subtracts the cube of $x$ and divides the result by your age.

## 'Are You Prepared?' Answers

1. $(-1, 3)$ 2. 21.5 3. $\{x \mid x \neq -4\}$ 4. $\{x \mid x < -1\}$




# 1.2 The Graph of a Function

**PREPARING FOR THIS SECTION** *Before getting started, review the following:*

- Graphs of Equations (Foundations Section 2, pp. 9–11)
- Intercepts (Foundations Section 2, pp. 11–12)

**Now Work** the **'Are You Prepared?'** problems on page 61.

**OBJECTIVES** 1 Identify the Graph of a Function (p. 57)
2 Obtain Information from or about the Graph of a Function (p. 58)

In applications, a graph often demonstrates more clearly the relationship between two variables than, say, an equation or table would. For example, Table 1 on the next page shows the average price of gasoline at a particular gas station in Texas (for the years 1983–2012 adjusted for inflation, based on 2012 dollars). If we plot these data and then connect the points, we obtain Figure 13 (also on the next page).