

STEP 1: $y = \sqrt{x}$

Square root function

STEP 2: $y = \sqrt{-x}$

Replace x by $-x$; reflect about the y -axis.

STEP 3: $y = \sqrt{-(x-1)} = \sqrt{1-x}$

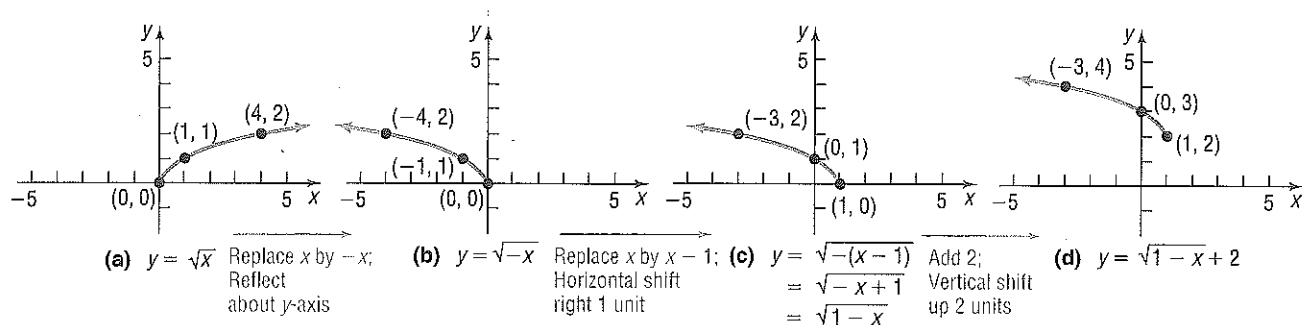
Replace x by $x-1$; horizontal shift to the right 1 unit.

STEP 4: $y = \sqrt{1-x} + 2$

Add 2; vertical shift up 2 units.

See Figure 57.

Figure 57

The domain of f is $(-\infty, 1]$ and the range is $[2, \infty)$.

Now Work PROBLEM 61

1.5 Assess Your Understanding

Concepts and Vocabulary

- Suppose that the graph of a function f is known. Then the graph of $y = f(x-2)$ may be obtained by a(n) _____ shift of the graph of f to the _____ a distance of 2 units.
- Suppose that the graph of a function f is known. Then the graph of $y = f(-x)$ may be obtained by a reflection about the _____-axis of the graph of the function $y = f(x)$.
- Suppose that the graph of a function g is known. The graph of $y = g(x) + 2$ may be obtained by a _____ shift of the graph of g _____ a distance of 2 units.
- True or False** The graph of $y = -f(x)$ is the reflection about the x -axis of the graph of $y = f(x)$.
- True or False** To obtain the graph of $f(x) = \sqrt{x+2}$, shift the graph of $y = \sqrt{x}$ horizontally to the right 2 units.
- True or False** To obtain the graph of $f(x) = x^3 + 5$, shift the graph of $y = x^3$ vertically up 5 units.

Skill Building

In Problems 7–18, match each graph to one of the following functions:

A. $y = x^2 + 2$

B. $y = -x^2 + 2$

C. $y = |x| + 2$

D. $y = -|x| + 2$

E. $y = (x-2)^2$

F. $y = -(x+2)^2$

G. $y = |x-2|$

H. $y = -|x+2|$

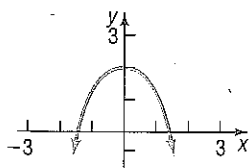
I. $y = 2x^2$

J. $y = -2x^2$

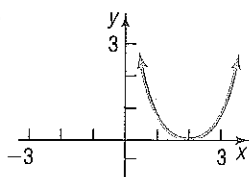
K. $y = 2|x|$

L. $y = -2|x|$

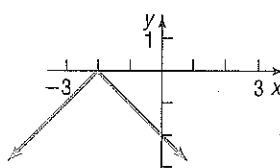
7.



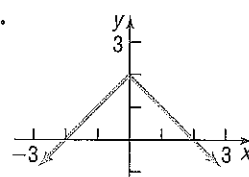
8.

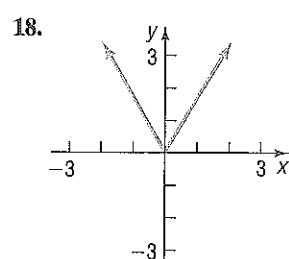
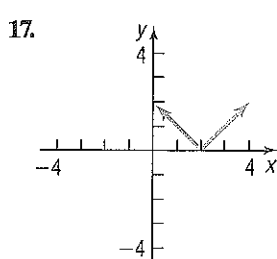
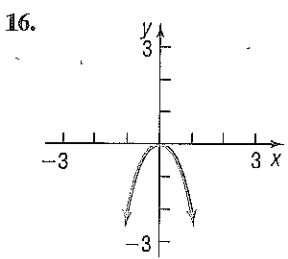
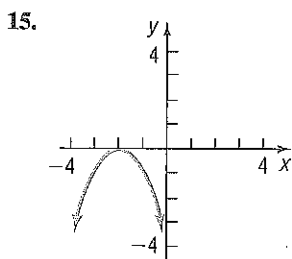
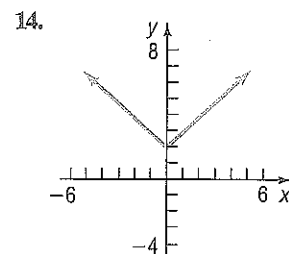
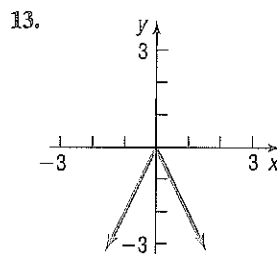
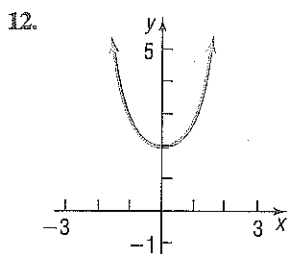
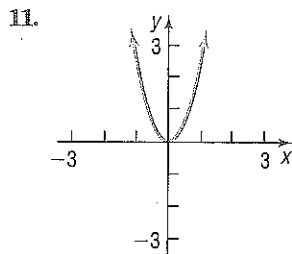


9.



10.





In Problems 19–26, write the function whose graph is the graph of $y = x^3$, but is:

- | | |
|---|---|
| 19. Shifted to the right 4 units | 20. Shifted to the left 4 units |
| 21. Shifted up 4 units | 22. Shifted down 4 units |
| 23. Reflected about the y -axis | 24. Reflected about the x -axis |
| 25. Vertically stretched by a factor of 4 | 26. Horizontally stretched by a factor of 4 |

In Problems 27–30, find the function that is finally graphed after each of the following transformations is applied to the graph of $y = \sqrt{x}$ in the order stated.

- | | |
|--|--|
| 27. (1) Shift up 2 units
(2) Reflect about the x -axis
(3) Reflect about the y -axis | 28. (1) Reflect about the x -axis
(2) Shift right 3 units
(3) Shift down 2 units |
| 29. (1) Reflect about the x -axis
(2) Shift up 2 units
(3) Shift left 3 units | 30. (1) Shift up 2 units
(2) Reflect about the y -axis
(3) Shift left 3 units |
| 31. If $(3, 6)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = -f(x)$?
(a) $(6, 3)$ (b) $(6, -3)$
(c) $(3, -6)$ (d) $(-3, 6)$ | 32. If $(3, 6)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = f(-x)$?
(a) $(6, 3)$ (b) $(6, -3)$
(c) $(3, -6)$ (d) $(-3, 6)$ |
| 33. If $(1, 3)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = 2f(x)$?
(a) $(1, 3)$ (b) $(2, 3)$
(c) $(1, 6)$ (d) $(\frac{1}{2}, 3)$ | 34. If $(4, 2)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = f(2x)$?
(a) $(4, 1)$ (b) $(8, 2)$
(c) $(2, -2)$ (d) $(4, 4)$ |
| 35. Suppose that the x -intercepts of the graph of $y = f(x)$ are -5 and 3 .
(a) What are the x -intercepts of the graph of $y = f(x + 2)$?
(b) What are the x -intercepts of the graph of $y = f(x - 2)$?
(c) What are the x -intercepts of the graph of $y = 4f(x)$?
(d) What are the x -intercepts of the graph of $y = f(-x)$? | 36. Suppose that the x -intercepts of the graph of $y = f(x)$ are -8 and 1 .
(a) What are the x -intercepts of the graph of $y = f(x + 4)$?
(b) What are the x -intercepts of the graph of $y = f(x - 3)$?
(c) What are the x -intercepts of the graph of $y = 2f(x)$?
(d) What are the x -intercepts of the graph of $y = f(-x)$? |
| 37. Suppose that the function $y = f(x)$ is increasing on the interval $(-1, 5)$.
(a) Over what interval is the graph of $y = f(x + 2)$ increasing?
(b) Over what interval is the graph of $y = f(x - 5)$ increasing?
(c) What can be said about the graph of $y = -f(x)$?
(d) What can be said about the graph of $y = f(-x)$? | 38. Suppose that the function $y = f(x)$ is decreasing on the interval $(-2, 7)$.
(a) Over what interval is the graph of $y = f(x + 2)$ decreasing?
(b) Over what interval is the graph of $y = f(x - 5)$ decreasing?
(c) What can be said about the graph of $y = -f(x)$?
(d) What can be said about the graph of $y = f(-x)$? |

In Problems 39–68, graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function (for example, $y = x^2$) and show all stages. Be sure to show at least three key points. Find the domain and the range of each function.

39. $f(x) = x^2 - 1$

42. $g(x) = x^3 - 1$

45. $f(x) = (x - 1)^3 + 2$

48. $g(x) = \frac{1}{2}\sqrt{x}$

51. $f(x) = -\sqrt[3]{x}$

54. $g(x) = \frac{1}{-x}$

57. $f(x) = 2(x + 1)^2 - 3$

60. $g(x) = 3|x + 1| - 3$

63. $f(x) = -(x + 1)^3 - 1$

66. $g(x) = 4\sqrt{2 - x}$

40. $f(x) = x^2 + 4$

43. $h(x) = \sqrt{x + 2}$

46. $f(x) = (x + 2)^3 - 3$

49. $h(x) = \frac{1}{2x}$

52. $f(x) = -\sqrt{x}$

55. $h(x) = -x^3 + 2$

58. $f(x) = 3(x - 2)^2 + 1$

61. $h(x) = \sqrt{-x} - 2$

64. $f(x) = -4\sqrt{x - 1}$

67. $h(x) = 2 \operatorname{int}(x - 1)$

41. $g(x) = x^3 + 1$

44. $h(x) = \sqrt{x + 1}$

47. $g(x) = 4\sqrt{x}$

50. $h(x) = \sqrt[3]{2x}$

53. $g(x) = \sqrt[3]{-x}$

56. $h(x) = \frac{1}{-x} + 2$

59. $g(x) = 2\sqrt{x - 2} + 1$

62. $h(x) = \frac{4}{x} + 2$

65. $g(x) = 2|1 - x|$

68. $h(x) = \operatorname{int}(-x)$

In Problems 69–72, the graph of a function f is illustrated. Use the graph of f as the first step toward graphing each of the following functions:

(a) $F(x) = f(x) + 3$

(b) $G(x) = f(x + 2)$

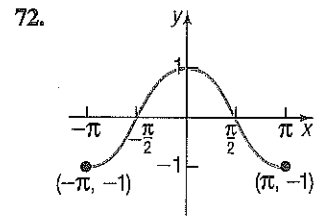
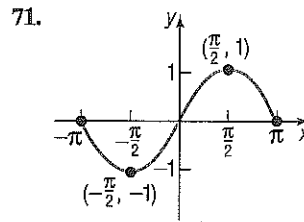
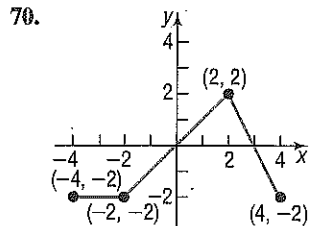
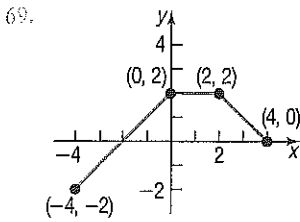
(c) $P(x) = -f(x)$

(d) $H(x) = f(x + 1) - 2$

(e) $Q(x) = \frac{1}{2}f(x)$

(f) $g(x) = f(-x)$

(g) $h(x) = f(2x)$



Mixed Practice

In Problems 73–82, complete the square of each quadratic expression. Then graph each function using the technique of shifting. (If necessary, refer to Appendix A, Section A.4 to review completing the square.)

73. $f(x) = x^2 + 2x$

74. $f(x) = x^2 - 6x$

75. $f(x) = x^2 - 8x + 1$

76. $f(x) = x^2 + 4x + 2$

77. $f(x) = x^2 + x + 1$

78. $f(x) = x^2 - x + 1$

79. $f(x) = 2x^2 - 12x + 19$

80. $f(x) = 3x^2 + 6x + 1$

81. $f(x) = -3x^2 - 12x - 17$

82. $f(x) = -2x^2 - 12x - 13$

83. (a) Graph $f(x) = |x - 3| - 3$ using transformations.

(b) Find the area of the region bounded by f and the x -axis that lies below the x -axis.

84. (a) Graph $f(x) = -2|x - 4| + 4$ using transformations.

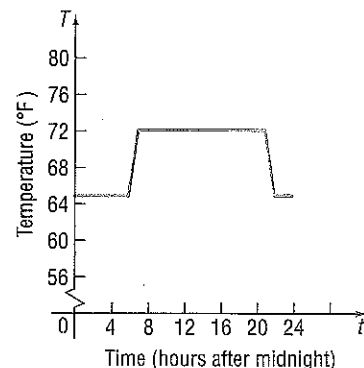
(b) Find the area of the region bounded by f and the x -axis that lies above the x -axis.

Applications and Extensions

85. The equation $y = (x - c)^2$ defines a family of parabolas, one parabola for each value of c . On one set of coordinate axes, graph the members of the family for $c = 0$, $c = 3$, and $c = -2$.

86. Repeat Problem 85 for the family of parabolas $y = x^2 + c$.

87. **Thermostat Control** Energy conservation experts estimate that homeowners can save 5% to 10% on winter heating bills by programming their thermostats 5 to 10 degrees lower while sleeping. In the given graph, the temperature T (in degrees Fahrenheit) of a home is given as a function of time t (in hours after midnight) over a 24-hour period.



- (a) At what temperature is the thermostat set during daytime hours? At what temperature is the thermostat set overnight?
- (b) The homeowner reprograms the thermostat to $y = T(t) - 2$. Explain how this affects the temperature in the house. Graph this new function.
- (c) The homeowner reprograms the thermostat to $y = T(t + 1)$. Explain how this affects the temperature in the house. Graph this new function.

Source: Roger Albright, *547 Ways to Be Fuel Smart*, 2000

- 88. Digital Music Revenues** The total projected worldwide digital music revenues R , in millions of dollars, for the years 2012 through 2017 can be estimated by the function

$$R(x) = 28.6x^2 + 300x + 4843$$

where x is the number of years after 2012.

- (a) Find $R(0)$, $R(3)$, and $R(5)$ and explain what each value represents.
- (b) Find $r = R(x - 2)$.
- (c) Find $r(2)$, $r(5)$, and $r(7)$ and explain what each value represents.
- (d) In the model r , what does x represent?
- (e) Would there be an advantage in using the model r when estimating the projected revenues for a given year instead of the model R ?

Source: *IFPI Digital Music Report*, 2013

- 89. Temperature Measurements** The relationship between the Celsius ($^{\circ}\text{C}$) and Fahrenheit ($^{\circ}\text{F}$) scales for measuring temperature is given by the equation

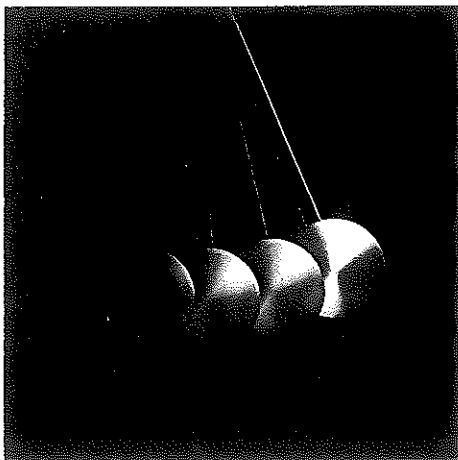
$$F = \frac{9}{5}C + 32$$

The relationship between the Celsius ($^{\circ}\text{C}$) and Kelvin (K) scales is $K = C + 273$. Graph the equation $F = \frac{9}{5}C + 32$ using degrees Fahrenheit on the y -axis and degrees Celsius on the x -axis. Use the techniques introduced in this section to obtain the graph showing the relationship between Kelvin and Fahrenheit temperatures.

- 90. Period of a Pendulum** The period T (in seconds) of a simple pendulum is a function of its length l (in feet) defined by the equation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where $g \approx 32.2$ feet per second per second is the acceleration of gravity.



- (a) Use a graphing utility to graph the function $T = T(l)$.
- (b) Now graph the functions $T = T(l + 1)$, $T = T(l + 2)$, and $T = T(l + 3)$.
- (c) Discuss how adding to the length l changes the period T .
- (d) Now graph the functions $T = T(2l)$, $T = T(3l)$, and $T = T(4l)$.
- (e) Discuss how multiplying the length l by factors of 2, 3, and 4 changes the period T .

- 91. Cigar Company Profits** The daily profits of a cigar company from selling x cigars are given by

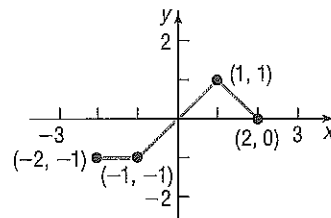
$$p(x) = -0.05x^2 + 100x - 2000$$

The government wishes to impose a tax on cigars (sometimes called a *sin tax*) that gives the company the option of either paying a flat tax of \$10,000 per day or a tax of 10% on profits. As chief financial officer (CFO) of the company, you need to decide which tax is the better option for the company.

- (a) On the same screen, graph $Y_1 = p(x) - 10,000$ and $Y_2 = (1 - 0.10)p(x)$.
- (b) Based on the graph, which option would you select? Why?
- (c) Using the terminology learned in this section, describe each graph in terms of the graph of $p(x)$.
- (d) Suppose that the government offered the options of a flat tax of \$4800 or a tax of 10% on profits. Which would you select? Why?

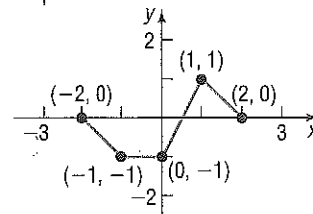
- 92.** The graph of a function f is illustrated in the figure.

- (a) Draw the graph of $y = |f(x)|$.
- (b) Draw the graph of $y = f(|x|)$.



- 93.** The graph of a function f is illustrated in the figure.

- (a) Draw the graph of $y = |f(x)|$.
- (b) Draw the graph of $y = f(|x|)$.



- 94.** Suppose $(1, 3)$ is a point on the graph of $y = f(x)$.
- (a) What point is on the graph of $y = f(x + 3) - 5$?
- (b) What point is on the graph of $y = -2f(x - 2) + 1$?
- (c) What point is on the graph of $y = f(2x + 3)$?
- 95.** Suppose $(-3, 5)$ is a point on the graph of $y = g(x)$.
- (a) What point is on the graph of $y = g(x + 1) - 3$?
- (b) What point is on the graph of $y = -3g(x - 4) + 3$?
- (c) What point is on the graph of $y = g(3x + 9)$?