

EXAMPLE 12**Finding the Slope of a Line Perpendicular to Another Line**

If a line has slope $\frac{3}{2}$, any line having slope $-\frac{2}{3}$ is perpendicular to it.

EXAMPLE 13**Finding the Equation of a Line Perpendicular to a Given Line**

Find an equation of the line that contains the point $(1, -2)$ and is perpendicular to the line $x + 3y = 6$. Graph the two lines.

Solution

First write the equation of the given line in slope-intercept form to find its slope.

$$x + 3y = 6$$

$$3y = -x + 6 \quad \text{Proceed to solve for } y.$$

$$y = -\frac{1}{3}x + 2 \quad \text{Place in the form } y = mx + b.$$

The given line has slope $-\frac{1}{3}$. Any line perpendicular to this line will have slope 3.

Because the point $(1, -2)$ is on this line with slope 3, use the point-slope form of the equation of a line.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - (-2) = 3(x - 1) \quad m = 3, x_1 = 1, y_1 = -2$$

To obtain other forms of the equation, proceed as follows:

$$y + 2 = 3(x - 1)$$

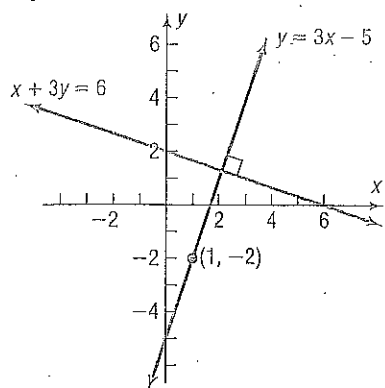
$$y + 2 = 3x - 3 \quad \text{Simplify.}$$

$$y = 3x - 5 \quad \text{Slope-intercept form}$$

$$3x - y = 5 \quad \text{General form}$$

Figure 44 shows the graphs.

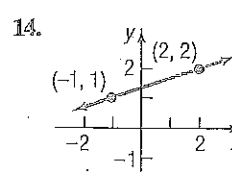
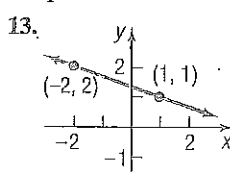
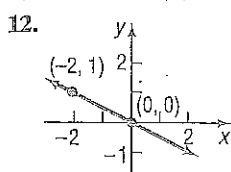
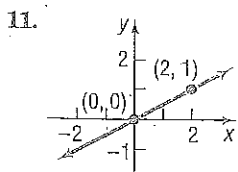
 **Now Work** PROBLEM 65

Figure 44**F.3 Assess Your Understanding****Concepts and Vocabulary**

- The slope of a vertical line is _____; the slope of a horizontal line is _____.
- For the line $2x + 3y = 6$, the x -intercept is _____ and the y -intercept is _____.
- A horizontal line is given by an equation of the form _____, where b is the _____.
- True or False** Vertical lines have an undefined slope.
- True or False** The slope of the line $2y = 3x + 5$ is 3.
- True or False** The point $(1, 2)$ is on the line $2x + y = 4$.
- Two nonvertical lines have slopes m_1 and m_2 , respectively. The lines are parallel if _____ and the _____ are unequal; the lines are perpendicular if _____.
- The lines $y = 2x + 3$ and $y = ax + 5$ are parallel if $a =$ _____.
- The lines $y = 2x - 1$ and $y = ax + 2$ are perpendicular if $a =$ _____.
- True or False** Perpendicular lines have slopes that are reciprocals of one another.

Skill Building

In Problems 11–14, (a) find the slope of the line and (b) interpret the slope.



In Problems 15–22, plot each pair of points and determine the slope of the line containing them. Graph the line.

- $(2, 3); (4, 0)$
- $(4, 2); (3, 4)$
- $(-2, 3); (2, 1)$
- $(-1, 1); (2, 3)$
- $(-3, -1); (2, -1)$
- $(4, 2); (-5, 2)$
- $(-1, 2); (-1, -2)$
- $(2, 0); (2, 2)$

In Problems 23–30, graph the line containing the point P and having slope m .

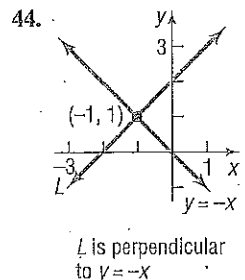
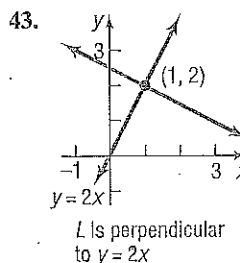
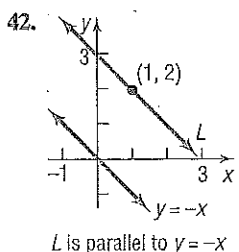
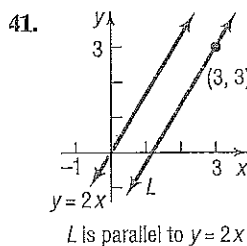
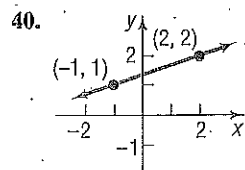
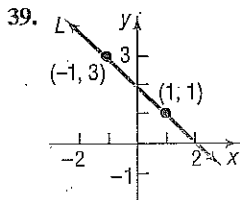
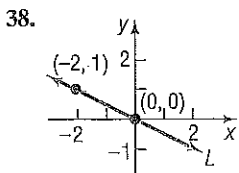
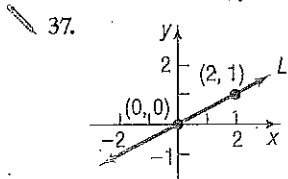
23. $P = (1, 2)$; $m = 3$ 24. $P = (2, 1)$; $m = 4$ 25. $P = (2, 4)$; $m = -\frac{3}{4}$ 26. $P = (1, 3)$; $m = -\frac{2}{5}$
 27. $P = (-1, 3)$; $m = 0$ 28. $P = (2, -4)$; $m = 0$ 29. $P = (0, 3)$; slope undefined 30. $P = (-2, 0)$; slope undefined

In Problems 31–36, the slope and a point on a line are given. Use this information to locate three additional points on the line. Answers may vary.

[Hint: It is not necessary to find the equation of the line. See Example 2.]

31. Slope 4; point $(1, 2)$ 32. Slope 2; point $(-2, 3)$ 33. Slope $-\frac{3}{2}$; point $(2, -4)$
 34. Slope $\frac{4}{3}$; point $(-3, 2)$ 35. Slope -2 ; point $(-2, -3)$ 36. Slope -1 ; point $(4, 1)$

In Problems 37–44, find an equation of the line L .



In Problems 45–70, find an equation for the line with the given properties. Express your answer using either the general form or the slope-intercept form of the equation of a line, whichever you prefer.

45. Slope = 3; containing the point $(-2, 3)$ 46. Slope = 2; containing the point $(4, -3)$
 47. Slope = $-\frac{2}{3}$; containing the point $(1, -1)$ 48. Slope = $\frac{1}{2}$; containing the point $(3, 1)$
 49. Containing the points $(1, 3)$ and $(-1, 2)$ 50. Containing the points $(-3, 4)$ and $(2, 5)$
 51. Slope = -3 ; y -intercept = 3 52. Slope = -2 ; y -intercept = -2
 53. x -intercept = 2; y -intercept = -1 54. x -intercept = -4 ; y -intercept = 4
 55. Slope undefined; containing the point $(2, 4)$ 56. Slope undefined; containing the point $(3, 8)$
 57. Horizontal; containing the point $(-3, 2)$ 58. Vertical; containing the point $(4, -5)$
 59. Parallel to the line $y = 2x$; containing the point $(-1, 2)$ 60. Parallel to the line $y = -3x$; containing the point $(-1, 2)$
 61. Parallel to the line $2x - y = -2$; containing the point $(0, 0)$ 62. Parallel to the line $x - 2y = -5$; containing the point $(0, 0)$
 63. Parallel to the line $x = 5$; containing the point $(4, 2)$ 64. Parallel to the line $y = 5$; containing the point $(4, 2)$
 65. Perpendicular to the line $y = \frac{1}{2}x + 4$; containing the point $(1, -2)$ 66. Perpendicular to the line $y = 2x - 3$; containing the point $(1, -2)$
 67. Perpendicular to the line $2x + y = 2$; containing the point $(-3, 0)$ 68. Perpendicular to the line $x - 2y = -5$; containing the point $(0, 4)$
 69. Perpendicular to the line $x = 8$; containing the point $(3, 4)$ 70. Perpendicular to the line $y = 8$; containing the point $(3, 4)$

In Problems 71–90, find the slope and y-intercept of each line. Graph the line.

71. $y = 2x + 3$

72. $y = -3x + 4$

73. $\frac{1}{2}y = x - 1$

74. $\frac{1}{3}x + y = 2$

75. $y = \frac{1}{2}x + 2$

76. $y = 2x + \frac{1}{2}$

77. $x + 2y = 4$

78. $-x + 3y = 6$

79. $2x - 3y = 6$

80. $3x + 2y = 6$

81. $x + y = 1$

82. $x - y = 2$

83. $x = -4$

84. $y = -1$

85. $y = 5$

86. $x = -2$

87. $y - x = 0$

88. $x + y = 0$

89. $2y - 3x = 0$

90. $3x + 2y = 0$

In Problems 91–100, (a) find the intercepts of the graph of each equation and (b) graph the equation.

91. $2x + 3y = 6$

92. $3x - 2y = 6$

93. $-4x + 5y = 40$

94. $6x - 4y = 24$

95. $7x + 2y = 21$

96. $5x + 3y = 18$

97. $\frac{1}{2}x + \frac{1}{3}y = 1$

98. $x - \frac{2}{3}y = 4$

99. $0.2x - 0.5y = 1$

100. $-0.3x + 0.4y = 1.2$

101. Find an equation of the x-axis.

102. Find an equation of the y-axis.

In Problems 103–106, the equations of two lines are given. Determine whether the lines are parallel, perpendicular, or neither.

103. $y = 2x - 3$
 $y = 2x + 4$

104. $y = \frac{1}{2}x - 3$
 $y = -2x + 4$

105. $y = 4x + 5$
 $y = -4x + 2$

106. $y = -2x + 3$
 $y = -\frac{1}{2}x + 2$

THINK In Problems 107–110, match each graph with the correct equation:

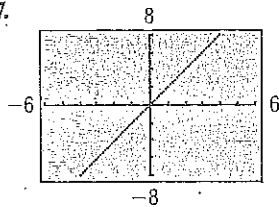
(a) $y = x$

(b) $y = 2x$

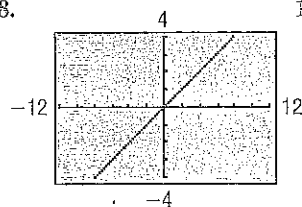
(c) $y = \frac{x}{2}$

(d) $y = 4x$

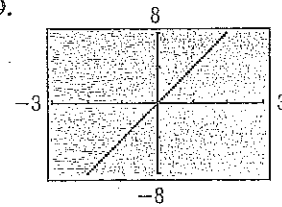
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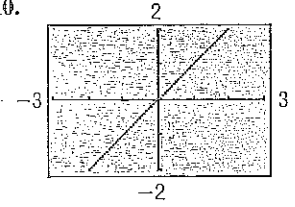
108.



109.



110.



Applications and Extensions

111. **Geometry** Use slopes to show that the triangle whose vertices are $(-2, 5)$, $(1, 3)$, and $(-1, 0)$ is a right triangle.
112. **Geometry** Use slopes to show that the quadrilateral whose vertices are $(1, -1)$, $(4, 1)$, $(2, 2)$, and $(5, 4)$ is a parallelogram.
113. **Geometry** Use slopes to show that the quadrilateral whose vertices are $(-1, 0)$, $(2, 3)$, $(1, -2)$, and $(4, 1)$ is a rectangle.
114. **Geometry** Use slopes and the distance formula to show that the quadrilateral whose vertices are $(0, 0)$, $(1, 3)$, $(4, 2)$, and $(3, -1)$ is a square.
115. **Truck Rentals** A truck rental company rents a moving truck for one day by charging \$39 plus \$0.60 per mile. Write a linear equation that relates the cost C , in dollars, of renting the truck to the number x of miles driven. What is the cost of renting the truck if the truck is driven 110 miles? 230 miles?
116. **Cost Equation** The **fixed costs** of operating a business are the costs incurred regardless of the level of production. Fixed costs include rent, fixed salaries, and costs of leasing machinery. The **variable costs** of operating a business are the costs that change with the level of output. Variable costs include raw materials, hourly wages, and

electricity. Suppose that a manufacturer of jeans has fixed daily costs of \$500 and variable costs of \$8 for each pair of jeans manufactured. Write a linear equation that relates the daily cost C , in dollars, of manufacturing the jeans to the number x of jeans manufactured. What is the cost of manufacturing 400 pairs of jeans? 740 pairs?

117. **Cost of Driving a Car** The annual fixed costs for owning a small sedan are \$6735, assuming the car is completely paid for. The cost to drive the car is approximately \$0.45 per mile. Write a linear equation that relates the cost C and the number x of miles driven annually.

Source: AAA, April 2012

118. **Wages of a Car Salesperson** Dan receives \$375 per week for selling new and used cars at a car dealership in Oak Lawn, Illinois. In addition, he receives 5% of the profit on any sales that he generates. Write a linear equation that represents Dan's weekly salary S when he has sales that generate a profit of x dollars.
119. **Electricity Rates in Illinois Commonwealth** Edison Company supplies electricity to residential customers for a monthly customer charge of \$13.04 plus 10.62 cents per kilowatt-hour for up to 800 kilowatt-hours (kw-hr).



- Write a linear equation that relates the monthly charge C , in dollars, to the number x of kilowatt-hours used in a month, $0 \leq x \leq 800$.
- Graph this equation.
- What is the monthly charge for using 200 kilowatt-hours?
- What is the monthly charge for using 500 kilowatt-hours?
- Interpret the slope of the line.

Source: Commonwealth Edison Company, January 2013.

- 120. Electricity Rates in Florida** Florida Power & Light Company supplies electricity to residential customers for a monthly customer charge of \$700 plus 8.55 cents per kilowatt-hour for up to 1000 kilowatt-hours (kw-hr).

- Write a linear equation that relates the monthly charge C , in dollars, to the number x of kilowatt-hours used in a month, $0 \leq x \leq 1000$.
- Graph this equation.
- What is the monthly charge for using 200 kilowatt-hours?
- What is the monthly charge for using 500 kilowatt-hours?
- Interpret the slope of the line.

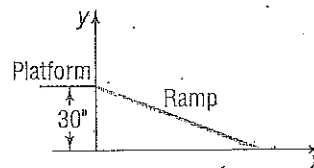
Source: Florida Power & Light Company, January 2013.

- 121. Measuring Temperature** The relationship between Celsius ($^{\circ}\text{C}$) and Fahrenheit ($^{\circ}\text{F}$) degrees of measuring temperature is linear. Find a linear equation relating $^{\circ}\text{C}$ and $^{\circ}\text{F}$ if 0°C corresponds to 32°F and 100°C corresponds to 212°F . Use the equation to find the Celsius measure of 70°F .

- 122. Measuring Temperature** The Kelvin (K) scale for measuring temperature is obtained by adding 273 to the Celsius temperature.

- Write a linear equation relating K and $^{\circ}\text{C}$.
- Write a linear equation relating K and $^{\circ}\text{F}$ (see Problem 121).

- 123. Access Ramp** A wooden access ramp is being built to reach a platform that sits 30 inches above the floor. The ramp drops 2 inches for every 25-inch run.



- Write a linear equation that relates the height y of the ramp above the floor to the horizontal distance x from the platform.
- Find and interpret the x -intercept of the graph of your equation.
- Design requirements stipulate that the maximum run be 30 feet and that the maximum slope be a drop of 1 inch for every 12 inches of run. Will this ramp meet the requirements? Explain.
- What slopes could be used to obtain the 30-inch rise and still meet design requirements?

Source: www.adaptiveaccess.com/wood_ramps.php

- 124. Cigarette Use** A report in the Child Trends DataBase indicated that in 2000, 20.6% of twelfth grade students reported daily use of cigarettes. In 2011, 10.3% of twelfth grade students reported daily use of cigarettes.

- Write a linear equation that relates the percent y of twelfth grade students who smoke cigarettes daily to the number x of years after 2000.
- Find the intercepts of the graph of your equation.
- Do these intercepts have any meaningful interpretation?
- Use your equation to predict the percent for the year 2025. Is this result reasonable?

Source: www.childtrends.org/databank

- 125. Product Promotion** A cereal company finds that the number of people who will buy one of its products in the first month that the product is introduced is linearly related to the amount of money it spends on advertising. If it spends \$40,000 on advertising, then 100,000 boxes of cereal will be sold, and if it spends \$60,000, then 200,000 boxes will be sold.

- Write a linear equation that relates the amount A spent on advertising to the number x of boxes the company aims to sell.
- How much expenditure on advertising is needed to sell 300,000 boxes of cereal?
- Interpret the slope.

- 126.** Show that the line containing the points (a, b) and (b, a) , $a \neq b$, is perpendicular to the line $y = x$. Also show that the midpoint of (a, b) and (b, a) lies on the line $y = x$.

- 127.** The equation $2x - y = C$ defines a family of lines, one line for each value of C . On one set of coordinate axes, graph the members of the family when $C = -4$, $C = 0$, and $C = 2$. Can you draw a conclusion from the graph about each member of the family?

- 128.** Prove that if two nonvertical lines have slopes whose product is -1 , then the lines are perpendicular. [Hint: Refer to Figure 43 and use the converse of the Pythagorean Theorem.]