

How do I solve Rational Eqs?

Rational Eqs have variables that are in fractions.

$$\text{ex) } \frac{1}{x} + \frac{1}{2} = \frac{3}{x+2}$$

Reminder: denominator $\neq 0$

Extraneous solus: any soln that raises denominator to be zero is extraneous.

Remember: +/- fractions

$$\text{algorithm: } \frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm cb}{bd}$$

$$\text{ex) } \frac{1}{5} + \frac{2}{3} \quad (\text{2 ways})$$

Common Denom.

$$\frac{1}{5} + \frac{2}{3} \quad \text{CD: 15}$$

$$\frac{3}{3} \cdot \frac{1}{5} + \frac{2}{3} \cdot \frac{5}{5}$$

$$\frac{3}{15} + \frac{10}{15}$$

$$\frac{13}{15}$$

algorithm

$$\frac{a}{b} \pm \frac{c}{d}$$

$$(1)(3) + (2)(5)$$

$$\frac{13}{15}$$

$$\frac{3 + 10}{15}$$

$$\frac{13}{15}$$

Methods to Solve

- ① Convert to a common denominator \rightarrow equate the numerator
- ② Multiply through by the common denominator
- ③ Cross multiply

Method 1

Solve $\frac{3}{x-2} - \frac{1}{x} = \frac{1}{5x}$

What is the common denominator for $x+2$, x , and $5x$?

$$\left. \begin{array}{l} x+2 = (x+2)(1) \\ x = \cancel{x} \cdot 1 \\ 5x = \cancel{5}x \end{array} \right\} \text{CD} = 5x(x+2)$$

extr soln

$$\begin{array}{l} x+2 \neq 0 \\ \boxed{x \neq -2} \\ \boxed{x \neq 0} \end{array}$$

$$\begin{array}{l} 5x \neq 0 \\ \cancel{5} \\ x \neq 0 \end{array}$$

$$\frac{5x}{5x} \cdot \frac{3}{x+2} - \frac{1}{x} \cdot \frac{5(x+2)}{5(x+2)} = \frac{1}{5x} \cdot \frac{(x+2)}{(x+2)}$$

$$\frac{(5x)(3)}{\cancel{5x}(x+2)} - \frac{5(x+2)}{\cancel{5x}(x+2)} = \frac{(x+2)}{\cancel{5x}(x+2)}$$

equating numerator

$$(5x)(3) - 5(x+2) = (x+2)$$

$$15x - 5x - 10 = x + 2$$

$$10x - 10 = x + 2$$

$$-x + 10 = x + 2$$

$$\rightarrow \frac{9x}{9} = \frac{12}{9}$$

$$x = \frac{12}{9} = \frac{4}{3}$$

Method 2: multiply thru by CD

$$\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x} \quad \text{CD: } 5x(x+2)$$

$$\frac{\cancel{5x(x+2)}}{1} \cdot \frac{3}{\cancel{x+2}} - \frac{1}{\cancel{x}} \cdot \frac{\cancel{5x(x+2)}}{1} = \frac{1}{\cancel{5x}} \cdot \frac{\cancel{5x(x+2)}}{1}$$

$$(5x)(3) - (5)(x+2) = (1)(x+2)$$

$$15x - 5x - 10 = x + 2$$

$$\begin{array}{r} 10x - 10 = x + 2 \\ -x + 10 \quad -x + 0 \\ \hline 9x = 12 \end{array}$$

$$\frac{9x}{9} = \frac{12}{9}$$

$$x = \frac{4}{3}$$

possible Ext. Soln

$$x=0, x=-2$$

Method 3: cross multiply

$$\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$$

Possible Ext. Solns

$$x=0, x=-2$$

First: $\frac{a}{b} \frac{3}{x+2} - \frac{c}{d} \frac{1}{x} = \frac{ad - cb}{bd}$

$$\frac{(3)(x) - (x+2)}{x(x+2)} = \frac{3x - x - 2}{x(x+2)} = \frac{2x-2}{x(x+2)}$$

Next: $\frac{2x-2}{x(x+2)} = \frac{1}{5x}$

Now cross multiply:

$$5x(2x-2) = x(x+2)$$

$$10x^2 - 10x = x^2 + 2x$$
$$\begin{array}{r} -x^2 - 2x \\ \hline 9x^2 - 12x = 0 \end{array}$$

$$9x^2 - 12x = 0$$

$$3x(3x-4) = 0$$

2 PP

$$3x = 0$$

$$3x = 0$$
$$\frac{3x}{3} = \frac{0}{3}$$

~~extraneous~~
 ~~$x=0$~~

$$3x - 4 = 0$$

$$3x - 4 = 0$$

$$\begin{array}{r} +4 \quad +4 \\ \hline 3x = 4 \end{array}$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$x = \frac{4}{3}$$

